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## NATIONAL UNIVERSITY OF UZBEKISTAN ASTRONOMICAL INSTITUTE

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# CCD-PHOTOMETRY OF SELECTED GRAVITATIONALLY LENSED QUASARS AND THE WIDE FIELDS CASES

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#### **INTRODUCTION**

**Topicality and relevance of the theme of the dissertation.** At the present time study of objects associated to the phenomenon of gravitational lensing, and especially gravitationally lensed quasars (GLQ) is one of the most important tasks of the modern observational cosmology. Such objects act as a giant natural telescope, whose resolution far exceeds the capabilities of modern ground-base and space instruments. To solve many problems of GLQ physics, first of all, it is necessary to study the time delays between the images of the lensed quasars and the various microlensing events occurring in them, which can be obtained only by their photometry. Determination of the time delay between the lensed components is directly related to the Hubble constant, the red-shifts of the source and the lens, and the gravitational potential of the latter. Microlensing also allows us to detect massive and invisible in any other cases objects or matter clusters, mainly in the form of compact objects. Hence the relevance of the thesis follows, since the initial stages of the origin of galaxies and the structure of dark matter can be observed in detail only through gravitational lensing.

Recently, in the field of gravitational lensing, a number important discoveries have been made by lead researchers. Particularly, a portion of dark matter in the form of low-mass stars in our Galaxy was identified, one of the smallest extrasolar planets was detected. The most massive large-scale structures in our Universe were detected, a direct empirical proof of the existence of dark matter was found. The measurements of the Hubble constant have been made. The most distant quasars with red-shift z > 7 were found, more than 150 GLQ were revealed on the base of observations.

In our Republic, great attention is also paid to the observational and theoretical aspects of the study of gravitational lensing. The directions of these fundamental studies, which have a great importance for the development of science in our country, are related to the Strategy of Actions for the further development of the Republic of Uzbekistan for 2017-2021. Thus, in the Maidanak Observatory

active monitoring observations of promising GLQ are carried out during last 20 years, a huge unique observational material on the basis of long-term homogeneous monitoring has been accumulated, the analysis of the light curves of the lensed images of a number of GLQ has been carried out, the exact expressions for deflection angle of light rays due to weak lensing around a black hole has been obtained. However, at the same time there are some unsolved issues, in particular, still unknown acceptable values of the amplitude and duration of the variability caused by microlensing in the objects of study of the thesis, unknown nature of the microlensing, sometimes there is even uncertainty in determination the red-shift of the lensing bodies, the cases of images of wide fields still has not been studied.

This dissertation work, to a certain extent, serves as an implemention of tasks in accordance with state normative documents, Decrees of the President of the Republic of Uzbekistan #PF-4947 "On the Strategy for the further development of the Republic of Uzbekistan" dated February 7, 2017, as well as the "Road map of the main directions of structural reforms in Uzbekistan for 2019-2021", published by the government of the Republic of Uzbekistan on November 29, 2018.

**Conformity of the research to the main priorities of science and technology development of the Republic.** The dissertation research has been carried out in accordance with the priority areas of science and technology in the Republic of Uzbekistan: II. "Power, energy and resource saving".

**Review of international scientific researches on dissertation subject.** The study of the phenomenon of gravitational lensing in general and the lensed quasars in particular, their detection, observations in different pass bands of electromagnetic radiation, processing of observational data and their interpretation has been carrying out in the leading research centers and universities, such as: The Sternberg Astronomical Institute (Russia), Astronomical Calculation Institute and Center of Astronomy of the University of Heidelberg, Leibniz Institute for Astrophysics Potsdam, The Max Planck Institute for Extraterrestrial Physics

(Germany), Laboratory of Astrophysics, Ecole Polytechnique Fédérale de Lausanne, The International Space Science Institute (Switzerland), The Institute for Computational Cosmology at Durham University, The Institute of Astronomy at Cambridge University (UK), The Warsaw University Observatory (Poland), Institute for Space Research (Netherlands), Institute for Radiophysics and Electronics of the National Academy of Sciences of Ukraine, Astronomical Institute of the Kharkiv National University (Ukraine), The Institute of Astrophysics and Geophysics of University of Liege (Belgium), Research Center for the Early Universe, Department of Physics, University of Tokyo, Subaru Telescope, National Astronomical Observatory of Japan (Japan), Department of modern physics at the University of Cantabria (Spain), Space Telescope Science Institute (USA), Ulugh Beg Astronomical Institute, National University of Uzbekistan (Uzbekistan).

On the study of gravitationally lensed quasars at the world level a number of scientific results have been obtained, including: the proportion of dark matter in the form of low-mass stars in our Galaxy was determined through observations of stars in the Magellanic Clouds, microlensing analysis detected one of the smallest known exoplanets, which suggests that cool, sub-Neptune-mass planets may be more common than gas giant planets (Center for Particle Astrophysics, University of California, Institut d'Astrophysique de Paris). Statistical analysis of the ellipticity of galaxies over large areas revealed massive large-scale structures in the Universe, Hubble telescope observations of two merging clusters and gravitational lensing maps allowed to find direct empirical proof of the existence of dark matter (The Canadian Institute for Theoretical Astrophysics; Kavli Institute for Particle Astrophysics and Cosmology). Several unique programs for the systematic detecting of gravitational lenses have been developed, the list of candidates and observationally confirmed GLQ is continuously updated by results of spectroscopic and optical observations (Subaru Telescope, National Astronomical Observatory of Japan; Institute of Astronomy, University of Cambridge;

Argelander-Institut für Astronomie, Universität Bonn). A large array of observational data was obtained during the COSMOGRAIL program and an effective photometric programs for image reconstruction were developed (Laboratoire d'Astrophysique, École Polytechnique Fédérale de Lausanne, Institut d'Astrophysique et de Géophysique, Université de Liège, Astronomical Institute of the Academy of Sciences of Uzbekistan (UBAI)). Independent photometric methods and computer programs have been developed (Sternberg Astronomical Institute; Astronomical Institute of the Kharkiv National University).

Currently, theoretical and observational studies of gravitationally lensed quasars in the world have been carried out in a number of priority areas, including: search for new GLQ based on statistical, spectral and photometric analysis of GAIA, SDSS and other databases; detecting of microlensing events, calculation of the time delay values and calculation of the Hubble constant; testing of models of the Universe and its large-scale units; measurement of the mass of galaxy clusters; solving the problems of ambiguity in gravitational lens models and influence of source position transformation on time delays at the University of Chicago (USA), Max-Planck-Institute fur Astrophysik (Garching, Germany), Early Universe Research Center (Tokyo, Japan).

**Degree of studiedness of the problem.** Currently a number of scientists from leading scientific centers around the world, such as Bikmaev I., Sakhibullin N. (Russia), Shalyapin V., Zheleznyak A., Sergeev A. (Ukraine), Burud I., Cohen J., Schechter P., Beuzit J.-L., Kochanek C. (USA), P.Magain, Sluse D. (Belgium), Courbin F., Dye S., Meylan G. (Switzerland), Jacobson P., Hjorth J. (Denmark) have carried out a complex search on monitoring observations of the new gravitational lensed quasars, the photometric study of double GLQ SBS1520+530 and FBQ0951+2635, study of the lensing properties of galaxies and modeling of these systems, calculation of time delay as well as detection of the microlensing. However, for these GLQ long-term optical observations have not yet been carried

out, and accordingly the phenomena of microlensing on a large time interval have not been studied, neither their amplitude nor duration have been known.

The determination of the X-ray flux of galaxy clusters in the direction of quadrupole component GLQs PG1115+080 and B1422+231, their infrared observations and the allocation of substructures of lensing galaxies, the determination of their red-shift, the analysis of emission properties in the spectrum of the GLQ H1413+117, the construction of models of all these systems have been provided by many scientists, such as Grant C., Bautz M., Chartas G., Garmire G., Tonry J., Eracleous M., Dai X., Agol E., Gallagher S. (USA), Chiba M., Minezaki T., Kashikawa N., Kataza H. (Japan), Hutsemekers D., B. Borguet (Belgium) and others. However, despite this efforts, long-term optical observations of these GLQ were not carried out, the time delays and the nature of microlensing were not fully determined. The problem of the red-shift of the lensing body in H1413+117 remained open, and B1422+231 was studied even less, since there were not even seasonal observations yet.

The processing of digital images of wide fields from zenith observations in time-delay integration (TDI) mode is generally of a pioneering nature. The idea of building a zenith telescope with a liquid mirror was proposed by Canadian scientists (Borra E., Hickson P., Cabanac R., Content R., Gibson B.,), and the implementation of this project in practice was carried out by the Belgian (Surdej J., Swings J.-P., DeBecker M., Delchambre L., Finet F.), Indian (Kumar B., Pradhan B., Sagar R., Pandey K., Anupama G.) and Polish (Bartczak P.) astronomers. But in their analysis of wide fields images work were not fully touched upon, it was not clear what factors will affect the astrometric and photometric calibration of observational data.

Connection of the topic of the dissertation to the scientific works of higher educational and research institutions, where the dissertation is carried out. The thesis was carried out at the Astronomical Institute of Academy of Sciences of Uzbekistan and the National University Uzbekistan within the scientific research projects for 2008-2018: F-F2-F058+061 "Study of gravitational lenses, forming galaxies and generalized models of astrophysical objects" (2007-2011), FA-EOF2-F03 "Development of photometric program for digital astronomical images and analysis of the effects of General relativity in the gravitational lensing" (2010-2011), F2-FA-F029 "Physics of the gravitational lenses, compact astrophysical objects and non-stationary disk systems" (2012-2016), VA-FA-F2-007 - "Investigation of the lensing effects of the selected galaxies-quasar in the optical band" (2017-2018).

The aim of the research is to study the physics of phenomena of formation of microlensing and the effect of delay time via CCD photometry of selected double and quadrupole gravitationally lensed quasars, as well as the development of algorithms for processing of the digital wide fields images.

The following tasks of the research were formulated to achieve this aims:

to develop the method for photometry of point sources in dense fields;

to determine the time of delays between the lensed components of the studied GLQ and dependences in the systems parameters;

to calculate the red-shift for the lens in the H1413+117;

to allocate an internal variability of the quasars-sources from microlensing events, identifying their mechanisms, clarification of the duration of the microlensing;

to determine microlensing types depending on their rate and duration, calculation of the lower limit of the mass for microlenses;

to develop the methods and algorithms for processing of digital images in TDI mode, analysis of factors affecting to the accuracy of photometric and astrometric measurements.

**The objects of the research** are gravitationally lensed quasars: SBS1520+530, FBQ0951+560, PG1115+080, H1413+117 and B1422+261.

The subjects of the research are observational data in the form of CCD images, light curves of the lensed components of the objects of the study, as well as images obtained in the time delay integration mode.

The methods of the research. Well-known methods of processing of the digital astronomical images, methods of analysis of time series, minimization methods, methods of constructing of models of gravitationally lensed systems were used.

### The scientific novelty of the research are as follows:

two types of microlensing were determined in the double GLQs FBQ 0951+2635 and SBS1520+530: depending on the rate of brightness variation they can be background and strong, and depending on the duration they are: long-term or short-term;

for selected GLQs the light curves were plotted and new time values in three quadrupole GLQ PG1115+080, H1413+117, B1422+231 were determined;

it was found that flux ratio A1/A2in GLQ PG1115+080 varies not only in time, but also depends on the wavelength of the filter and the color index (V–I) of the lensed components versus brightness and time was found;

a strong microlensing in GLQ H413+117 is found and the lower limit for the mass of the body responsible for this microlensing was assessed as well as the new model with a singular isothermal ellipsoid for the gravitational lens is proposed, and the value of its red-shift has been specified;

the method and algorithm of computer processing of wide fields images have been developed, and also the effect of precession in the images obtained at zenithal observations in the TDI mode was found.

Practical results of the research are as follows:

there was shown the value of the time delay depends linearly on the angular size of the GLQ system – the maximum distance between the lensed components;

the mechanisms of microlensing in the objects of research are revealed and their classes are determined: the effect of long-term microlensing in the double GLQ SBS1520+530 and FBQ0951+2635 was detected, which lasting more than 10 years and it is shown long time-scale fluctuations dominate over the short-term microlensing events;

a new method for photometric processing of the point images in dense fields was developed;

the independent method of estimation of the error-bars of the photometric and time delay measurements has been obtained;

the light curves in GLQ B1422+231 were obtained for the first time, which allowed to find a long-term (more than 6 years) brightness variability of the source-quasar and the effect of microlensing;

detected microlensing events support the hypothesis about the general nature of quadrupole systems and the inevitability of micro-lensing events in them;

the algorithm of the processing of wide-field images is used for treatment of an images obtained from meridian observations;

the new method for generation of synthetic light curves was proposed.

The reliability of the results of the study is proved by using modern and wellknown methods of photometric processing of CCD images, time series analysis, calculation of time delays, determination of microlensing events, modeling of gravitationally lensed systems.

Scientific and practical significance of the research results. The scientific importance of the research results is determined, first of all, by the obtained observational data of the studied GLQ, which significantly supplemented the existing knowledge about them became the basis for their further replenishment and detailed study of other GLQ.

The practical significance of the research results lies in the fact that the proposed photometric method of adaptive fitting of the point scattering function (PSF) will help to save observational time on telescopes, since it can work even with defocused images of point sources. The values of delay time for three GLQ found by us will facilitate further detection of microlensing in these systems. The

method and algorithm for processing of wide filed images in TDI mode which we have developed used in the framework of the 4-m ILMT project. The results are included in the special master's courses of the Astronomical Department of NUUz: "Quasars and active galaxies", "Problems of cosmogony" for bachelor's degree and "Physics of gravitational lensing", "Processing of astronomical images" of the master's degree.

**Application of the research results.** Based on the study of the physics of microlensing phenomena and the effect of time delay in the selected double and quadrupole gravitationally lensed quasars, as well as the development of algorithms for processing wide-field images:

the results obtained during study of the nature and duration of microlensing in FBQ 0951+2635 and SBS1520+530 have been used by researchers (references in the scientific journals Astronomy & Astrophysics, 2018, 2012; Monthly Notices of the Royal Astronomical Society, 2013; Astronomy & Astrophysics, 2012; Astrophysics & Supplement Series, 2011; Astrophysics Journal, 2010) in the model analysis of strong gravitational lenses and in the calculation of cosmological parameters in the framework of the GLENDAMA project. The obtained results allowed to get an extensive database of lensed quasars and to determine the long-term microlensing events in the other double GLQ;

the new values of time delays between lensed components in the GLQ PG 1115+080, obtained during monitoring in Maidanak Observatory used in the peerreviewed journals (references in the Astronomy & Astrophysics, 2018, 2013; Monthly Notices of the Royal Astronomical Society, 2016; The Astrophysics Journal Supplement Series, 2015; Astronomical Reports, 2015) in re-computing the time delay in this system and the development of a new method of calculation of this quantity in the presence of microlensing. The obtained results allowed to study the structure of the environment around the lenses in sight-of-line and to better understand the effect of large-scale structure on the lens statistics and their parameters; our analysis of brightness variations of lensed images in the GLQ PG 1115+080 and discovery of microlensing in bright component A1/A2 are used in the peer-reviewed journals (references in the Monthly Notices of the Royal Astronomical Society, 2016, 2018; Astronomy & Astrophysics, 2015, 2018; The Astrophysics Journal, 2016; Astronomical Reports, 2015) in the analysis of the microlensing effect and the development of a new method that using time delay values models microlensing maps. The obtained results allowed to re-estimate the Hubble constant value, as well as to develop a new method of statistical analysis of the light curves of distant sources perturbed by microlensing by point and extended masses;

The results of our researches of brightness variations in the lensed components of GLQH1413+117, time delay values, determination of strong microlensing events, calculation of red-shift of lensing body in this system used in the peer-reviewed journals (references in the Astronomy & Astrophysics, 2018; Astronomy & Astrophysics, 2018; 2018arXiv181002624W; Nature, 2018) in estimation of ambiguities in models of gravitational lenses and impact on time delays of the source position transformation, the spectroscopic proof of discovery of new gravitational lens system GRAL113100-441959 and the model prediction of the time delay. Our data were also included in a new database of optically bright lensed quasars in the northern hemisphere, and were used in the analysis of the current state of astronomy in Uzbekistan.

The methods and algorithms developed by us for wide-field images processing are used in the framework of the international observation project 4-m ILMT (letter from Liege University dated January 15, 2019). The obtained results allowed to form a sequence of programs for processing images of wide fields and to find new objects of space debris.

**Approbation of the research results.** The research results were reported in the form of reports and tested in 5 International and 11 Regional scientific and practical conferences.

**Publication of the research results.** The main results of the thesis are published in 11 scientific papers in the peer-reviewed journals recommended by the Supreme Attestation Commission of the Republic of Uzbekistan for publication of the main scientific results of doctoral thesis and 23 abstracts in the proceedings of international and regional conferences.

**Volume and structure of the dissertation.** The thesis consists of an Introduction, five chapters, conclusion and a list of references. The thesis is presented in 188 pages.

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# CHAPTER I. MICROLENSING AND TIME DELAYS IN GRAVITATIONALLY LENSED QUASARS (GLQ)

### §1.1. Lensing and microlensing in GLQ

The universe we observe today is the result of more than 13 billions of years of evolution, the objects of which are represented by the widest wealth, diversity and complexity. Many of these objects exhibit extreme physical conditions: extremely high density, temperature and brightness that cannot be reproduced in any laboratory on Earth. Among other celestial objects, gravitationally lensed quasars occupy a special place, which combine a number of interesting features. Quasars by themselves are amazing objects. They emit as much energy as hundreds of galaxies like ours, but within a volume of space comparable to the size of the Solar system. Even more interesting are gravitationally lensed quasars. The phenomenon of gravitational lensing exhibits a very interesting and unusual properties: it creates a cosmic mirages, multiple images of background sources, and often acts as a giant natural telescope with resolution far exceeds the capabilities of modern astronomical instruments [13]. These features of gravitationally lensed quasars allow to apply them to solve a number of topical questions of modern cosmology, such as the problem of dark matter and dark energy, the structure of the Universe, the Hubble constant, the structure of quasars and others (see, for example, [13,14,15]). In order to answer the above problems, first of all information about the time delays and microlensing in the lensed quasars is needed.

As we can see, one of the most interesting and fascinating phenomena in space is the effect of gravitational lensing. Conventional lenses, which are used in magnifying loupes or glasses, cause light rays to change direction during refraction and collect them at a focal point (for example, in your eye). Gravitational lensing works in a similar way and is an effect of the general theory of relativity, however here the light is deflected due to gravity. The gravitational field of a massive object, propagating far in space, causes the light rays passing close to this object (and, accordingly, through its gravitational field) to bend and change their directions. The more massive the object, the stronger its gravitational field and, consequently, the greater the magnitude of the bending of the light beam - just as the denser matter of the optical lens leads to a greater refraction of light.

The term "gravitational lens" - GL, introduced in 1919, successfully approaches this phenomenon, when a powerful gravitational field of a massive body accumulate radiation from a far source in the focal semiaxis [16]. However, despite the analogy to the optical lens, there is a fundamental difference between them. Namely, if in the normal optical lens the deflection angle  $\hat{\alpha}$  is directly proportional to the impact parameter  $\hat{\alpha} \sim \xi$ , in the GL this ratio is inversely proportional to  $\hat{\alpha} \sim 1/\xi$  [14].

The correct formula for the deflection angle of the light was proposed by A. Einstein in 1915 in the framework of general relativity

$$\hat{\alpha} = \frac{2R_S}{\xi} = \frac{4GM}{c^2\xi} \tag{1.1}$$

where G - gravitational constant, M - the mass of the lensing body,  $R_S$  - Schwarzschild radius for the body with a given mass.

This formula near the edge of the Sun gives the light deflection  $\hat{\alpha} \approx 1.75''$ . However, this value is exactly twice the value of deflection angle derived by him earlier in the framework of the special theory of relativity and even earlier by J.G. Zoldner [17] in the framework of Newton's theory of gravitation. This dispute was solved relatively quickly in favor of the latest Einstein's calculations. In the course of the experiment to measure the position of the stars near the edge of the Sun during a total eclipse in 1919 the validity of general relativity and one of its consequences - gravitational lensing was proved [18].



Fig. 1.1. A typical illustration of the gravitational lensing phenomena [19].

The geometry of the light deflection in a typical gravitational lens system (GLS) is shown in Fig.1.1. Here O - observer, L - lensing body,  $\eta$  - distance from the source to the observer-lens optical axis. D<sub>s</sub>, D<sub>ds</sub>, D<sub>d</sub> are the distances from the observer and the source, from the lens and the source, from the observer and the lens, respectively. I - the source image.  $\xi$  - the distance from the lens to the deflection point (impact parameter).  $\theta$  and  $\beta$  are the angles between the optical axis and the image and the source, respectively.  $\alpha$  is the reduced angle, that is angle between the source and the image relative to the observer O.  $\hat{\alpha}$  is the actual angle of refraction. We consider the relationship between these parameters below.

As seen in Fig. 1.1. the light ray emanating from the source S is deflected by an angle  $\hat{\alpha}$  near the L and tends to the observer. In general, the angles in Fig.1.1 are vectors, since the deflection does not need to be radial, since in real gravitational lenses the mass is not concentrated at the point, but is distributed within a volume. In the case of small angles, relation called the *lens equation* is true

$$\vec{\beta} = \vec{\theta} - \vec{\alpha} \left( \vec{\theta} \right) \tag{1.2}$$

This is the fundamental equation of the theory of gravitational lensing and connects the position of the source  $\vec{\beta}$  with the positions of its lensed images  $\vec{\theta}$  and the deflection angle  $\vec{\alpha}$ . This equation is nonlinear in the general case, since it is possible to obtain a set of images with positions  $\vec{\theta}_i$  corresponding to one source with position  $\vec{\beta}$ .

The simplest solution of the lens equation can be obtained for a point lens, where the deflection is proportional to the mass, and the impact parameter  $\xi = \theta \cdot D_d$ . Then



$$\beta(\theta) = \theta - \frac{D_{ds}}{D_d D_s} \frac{4GM}{c^2 \theta}$$
(1.3)

**Fig.1.2.** The positions of the source S and its images  $I_+$  and L relative to point mass M. One of the images L is inside the Einstein's ring (dashed line), and the other outside the ring.



**Fig.1.3.** Einstein ring first observed by the Sloan Digital Sky Survey and then followed up with the Hubble Space Telescope. Foreground lens is a galaxy at the center, while the background object is almost perfectly aligned so is seen as a ring (ESA/Hubble & NASA [15]).

If we assume that the source is directly on the optical axis relative to the observer, i.e.  $\beta = 0$ , then its image appears as a ring which radius is

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{ds}}{D_d D_s}} = \sqrt{2R_s \frac{D_{ds}}{D_d D_s}}.$$
(1.4)

 $\theta_E$  is called the Einstein's ring radius. Thus, the expression (1.3) can be rewritten

$$\beta = \theta - \frac{\theta_E^2}{\theta}.$$
 (1.5)

Then there will appear two images with coordinates corresponding to the solution of the quadratic equation (1.5)

$$\theta_{\pm} = \frac{1}{2} \left( \beta \pm \sqrt{\beta^2 + 4\theta_E^2} \right). \tag{1.6}$$

The distance between them is  $\sqrt{\beta^2 + 4\theta_E^2}$ . The lens equation can also be expressed by the surface mass density

$$M(\xi) = 2\pi \int_{0}^{\xi} \Sigma(\vec{\xi}) \cdot \vec{\xi} \cdot d\vec{\xi}$$
(1.7)

where  $\Sigma(\vec{\xi})$  is the two-dimensional distribution of the mass in the lens.

In this case, the expression for the deflection angle for an arbitrary mass distribution can be written as follows:

$$\vec{\hat{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi}') \, \Sigma(\vec{\xi}')}{\left|\vec{\xi} - \vec{\xi}'\right|^2} d\vec{\xi}'.$$
(1.8)

The calculations are much simpler if we assume constant  $\Sigma(\vec{\xi})$ . Then the reduced deflection angle is rewritten as follows:

$$\alpha(\theta) = \frac{D_{ds}}{D_s} \frac{4G}{c^2 \xi} (\Sigma \pi \xi^2) = \frac{4G\pi}{c^2} \frac{D_d D_{ds}}{D_s} \theta.$$
(1.9)

Now we introduce the critical surface-mass density:

$$\Sigma_{cr} = \frac{c^2}{4G\pi} \frac{D_s}{D_d D_{ds}} = 0.35 \ gr \cdot sm^{-2} \left(\frac{D}{1 \ \text{Gpc}}\right)^{-1}$$
(1.10)

where  $D = \frac{D_d D_{ds}}{D_s}$  – effective distance. The critical surface-mass density is usually expressed in terms of the mass of the lens M "distributed" over the surface of the Einstein's ring:

$$\Sigma_{cr} = \frac{M}{R_E^2 \pi} \tag{1.11}$$

where  $R_E = \theta_E \cdot D_d$ . In the expressions for the surface mass densities, the deflection angle will be written:

$$\alpha(\theta) = \frac{\Sigma}{\Sigma_{cr}} \, \theta. \tag{1.12}$$

The critical surface mass density is very important in GL theory. Through it you can express the "strength" of gravitational lens. For example, the average surface mass density inside the Einstein's ring is always equal to the critical  $\Sigma_{cr}$ . In the case of a random mass distribution, the fulfillment of the condition  $\Sigma > \Sigma_{cr}$  at any point guarantees the appearance of several images [20, 21].

The next concept, which is widely used in the theory of gravitational lensing, is the amplification of a lens and its magnification. With gravitational lensing, the surface brightness of the source remains unchanged (since no new photons are produced), however, the visible solid angle of the source can change. By definition, the flux from an infinitesimal source is equal to the product of its brightness and its solid angle. The total flux obtained in the image of a gravitationally lensed source varies in proportion to the ratio between the solid angles of the image  $d\dot{\Omega}_I$  and the source  $d\dot{\Omega}_S$ . Therefore, magnification factor  $\mu$  can be written as:

$$\mu = \frac{image \ area}{source \ area} = \frac{d\dot{\Omega}_I}{d\dot{\Omega}_S} \tag{1.13}$$

If we know the angular coordinates of the source  $\vec{\beta}$  – and the image –  $\vec{\theta}$ , the magnification factor expressed as:

$$\mu = \frac{d\dot{\Omega}_I}{d\dot{\Omega}_S} = \left| \det \frac{d\beta}{d\theta} \right|^{-1}$$
(1.14)

This means that the change of the solid angle in GL is expressed by the Jacobi matrix of the mapping  $\vec{\theta} \rightarrow \vec{\beta}$ . For a lens with a spherically symmetric mass distribution, expression (1.14) is transformed

$$\mu = \frac{\theta}{\beta} \frac{d\theta}{d\beta} = \frac{\theta}{\beta} \left( \frac{1}{1 - d\alpha/d\theta} \right).$$
(1.15)

If several images of the source appear during lensing, then the ratios of the corresponding magnification factors are equal to the ratio of the flux from the images.

The determinant of Jacobi matrix can be both negative and positive, or tends to 0 (here  $\mu \rightarrow \infty$ ). Therefore, this can be said about the negative and positive parity of the images. In the plane of the lens there are regions in which the parity of the images can be of different polarity. Such regions are separated by critical curves, where the Jacobian tends to zero. When the *critical curves* are mapped onto the plane of the source, we get a caustic curve. Note that the requirement of  $\mu \rightarrow \infty$ on the critical curve does not mean that the brightness of the image increases to infinity, because actually the real sources have finite angular dimensions, which leads to a finite magnification.

The shape of critical and caustic lines depends on the mass distribution in GL. Thus, for a spherically symmetric mass distribution, the critical curves are circles that transform into a point after the mapping. And for an elliptical distribution, the critical curves are represented by ellipses. When they are mapped, a rhombic caustic will appear (Fig. 1.4).



**Fig. 1.4.** Critical curves (left) and caustics (right) for a lens with an elliptical mass distribution. The numbers in the right part of the figure indicate the number of images when the source falls into the corresponding area [20].

In the first decades after experimental confirmation of the validity of GL by F. Dyson et al., there were disputes about the probability of observation GLS. In this case, only lensing stars by stars was considered, since at that time only they were available for observation [22, 23]. It turned out the probability of observation of this phenomenon is negligible, since the angular distance between the lensed images is too small and they cannot be resolved by telescopes. However, consideration of lensing by galaxies have shown the probability of lensing is about  $\sim 1\%$  for sources with moderate redshifts [24,25] and the angular distance between images is quite accessible to be resolved by telescopes.

So the question arose - what objects will be lensed in this case. The discussions were speculative until quasars were discovered [26]. Quasars were a completely new class of objects and proved to be ideal sources in the study of the effects of gravitational lensing. They are most likely to be exposed to gravitational lensing not only because of cosmological distances, but also because they have giant luminosity and compact radiation regions in the optical range. The first gravitationally lensed quasar (GLQ) Q0957+561 was discovered in 1979. Thus, many years of disputes about the possibility of observing GLS were experimentally resolved and an observational part appeared in the study of

gravitational lensing. Since then, numerous effects of gravitational lensing were discovered.

Many researchers adhere to the following classification of observable types of GLS [20,21] - multiple lensing of quasars, microlensing of quasars, Einstein rings, giant arcs, weak lensing, microlensing in the Galaxy. Naturally, this classification is rather conditional, based, on the one hand, on the type of sources and lenses, and on the other hand on the magnification factor. Here we have deal to the first type of GLS - multiple lensing of quasars, and we also consider a physics of quasar microlensing.

Nowadays more than 150 observationally confirmed GLQ are known. Both lists of confirmed gravitationally lensing quasars and candidates for this type of objects are continuously published [27,28,29,30,31,32].

In terms of the number of lensed components, these systems can be either double, quadruple or more, and in shape, both symmetrical and asymmetric. However, in all cases there is a problem of identifying the presence of the GL effect. Long-term studies and observations in this field allowed to formulate a number of criteria for the membership of observable quasars to the GLQ: there are two or more point images with similar colors, redshifts of all images are the same or close to each other, the image spectra are identical or very close, there is a massive lensing body (usually a galaxy, less often a cluster of galaxies) between images, with a redshift much smaller than redshift of the images, the brightness variability of the source-quasar is reflected in all images taking into account the time delay time and other factors [19,20,21,29,30].

However, it is clear that only a few of the criteria are sufficient to identify the GLS. The spectra and colors of the lensed images may not be exactly identical because thee pass through various media (dust, nebula, etc.). Lens galaxy can be superimposed on the image and change its shape, or may be the brightness of the lens is too faint to be detected. The time of variation of a quasar brightness can be much shorter of the time delay. Therefore, it is very difficult to detect the gravitational lensing with absolute certainty.

The lensing considered above is caused by the gravity of massive galaxies and the distance between lensed images is of the order  $\sim 1''$  and more. This type of lensing called *macrolensing*. Another distinguishing feature of macrolensing is the fixed positions of the source and the lens and, as it follows, the invariance in time of the coordinates of the lensed images and the magnification factor [15].

However, there are situations where the relative position of the lens and the source varies in time. In these cases, the increase in visible brightness will also change with time. This phenomenon can be observed: if the brightness of the image is fixed at two points in time with different values of the fluxes, this difference can be caused by the displacement of the components of the GLQ relative to each other.



Fig. 1.5. Representation of the microlensing in a doubly imaged system [33].

How can it be? Let us imagine that at least one of the rays deflected by the lensing galaxy passes through it and/or its halo (Fig.1.5). In this case the light shall be affected by the gravitational fields of compact objects that make up the galaxy - planets, stars, black holes, etc., called microlenses. They will give at least one more

image of the source. Accordingly, this very phenomenon is called *microlensing*. The splitting angle at microlensing is  $\sim 4MG/D_Lc^2$ , then for a solar mass and a distance of the order of 1 Gpc it will be of the order of  $3 \times 10^{-6}$  angular seconds. Since such angles are very small, with the help of modern instruments it is not yet possible to see such microimages, since microscopic magnitudes are beyond the sensitivity of telescopes.

Microlensing will occur for sufficiently small masses and sufficiently remote lenses and sources. In this case we deal with compact objects of mass  $10^{-6} \le m/M_{\odot} \le 10^{6}$  and two distance scales-galactic, where the source and lens are at a distance of about 10 kpc, and cosmological, where distances of the order of Gpc.

This phenomenon was first discussed, apparently, in [34]. They found that, despite the great difference in the mass of the galaxy as a whole and of a single star these small objects can significantly affect the brightness of one of the observed macroimages. These variations are caused by time variations of the spatial configuration of the "source-quasar" - "galaxy-lens" - "microlens" - "observer". Later, many analytical and numerical studies of the microlensing phenomenon were carried out [35,36,37,38,39,40]. The observational evidence of galactic microlensing was first reported in 1993 [41-43] and since has been used to search compact dark matter (MACHO projects - "Massive Compact Halo Objects" and EROS – «Experience pour la Recherche d'Objets Sombres ") and extrasolar planets (OGLE project - "Optical Gravitational Lensing Experiment") in our Galaxy.

Extragalactic microlensing was observed in 1989, when individual objects in GLQ Q0957 + 561 and Q2237 + 0305 changed the brightness of one of the lenticular images relative to the other [44, 45]. Gravitational lensing affects the quasar radiation in two ways: the macrolensing by a galaxy with a mass of  $\sim 10^{12} M_{\odot}$  creates multiple images separated about one arcsecond. Since a galaxy contains the compact objects such as stars, brown dwarfs, planets and others, they can further divide the lensed images into unresolved "microimages" and affect

their brightness. The variability caused by microlenses can be used to study of two cosmological problems - the size and brightness profile of quasars, and the distribution of a compact dark matter along the line of sight.

Detection and observation of microlensing events is explained by the fact that objects of stellar mass can only increase sufficiently compact sources. The Einstein radius for microlenses is really small and can be comparable in size to the inner regions of quasars. Therefore, it is possible to observe a differential enhancement of the brightness of regions of different sizes, and since they have different colors, this causes a chromatic increase in brightness. Thus, if the flux ratios in the two spectral bands in the two (or more) images of the quasar are different, this may be a sign of microlensing. For example, the wide-line region of quasars is considered too large to undergo appreciable microlensing; Therefore, if the ratio of the linear stream of two images differs from the ratio of the optical flux, then there probably is microlensing in at least one of the images. Such microlensing is called chromatic.



**Fig. 1.6.** Microlensing-induced light curves for six minimum separations between the source and the lens. The separation is expressed in units of the Einstein radius [19].

Fig 1.6. an example is shown of increasing the gloss of a lensed image in a point lens depending on the relative position of the source. The microlensing characteristic curve offers an excellent perspective for detecting an object moving between us and the background source. For example, the lens of the solar mass in

our galaxy at a distance of 5 kpc from us will have a radius of the Einstein ring of the order of one angular second. If it moves at a speed of  $\sim 100$  km / s in a plane perpendicular to the line of sight, then within a year it will intersect the angular distance equal to the four Einstein's radius.

One of the fundamental principles of gravitational insight, as we noted above, is that the distortion of light rays does not depend on their wavelength. Therefore, these light curves should look the same at all wavelengths. This property is very useful in practice, since it is necessary to distinguish microlensing from objects with internal variability. Often, the last variability depends on the wavelength, so multicolor observations at different wavelengths is an effective tool for distinguishing microlensing from internal variability.

If the exact value of the time delay is known (for details, see §1.2.) the problem of detecting microlensing is much simplified. Indeed, if a certain amount of  $\Delta t$  on the light curve of the driven component relative to the leading one in one of them shows a noncorrelated light variation, then we can speak with more confidence about microlensing. So we detected changes in the light curves of the SBS1520+530, H1413+117 components caused by microlensing [7,46]. But this cannot always be done. Changes due to microlensing may be very weak. In this case it is necessary to use indirect methods, for example, chromatic microlensing.

The time of variation in the brightness of the components caused by microlensing depends on the transversal velocity of the source relative to the caustic. This velocity consists of three parts: the movement of the quasar, the motion of the lensing galaxy as a whole, and the motion of the observer [36]:

$$\vec{V} = \frac{\vec{v}_s}{1+z_s} - \frac{\vec{v}_d}{1+z_d} \frac{D_s}{D_d} + \frac{\vec{v}_0}{1+z_d} \frac{D_{ds}}{D_d}$$
(1.16)

where v- speed perpendicular to the line of sight, z - redshift, D - distance, and indices d, s, o - as usual lens, source and observer, respectively. In this expression,

the movements of individual microlenses in the lensing galaxy relative to the galaxy itself can be neglected.

If there is a well-established microlensing event, then one can draw conclusions about nature as a source-size and radial distribution of brightness, and microlenses-mass, density, transverse velocities. So in [40] a simple two-component model of a quasar source in the Einstein's Cross we considered, consisting of a bright and compact nucleus surrounded by a cold, extended shell. Analysis of the light curves of this GLQ allowed to make restrictions on the angular size and mass of the source –  $0.01 \div 0.1$  pc and  $0.01 \div 1 M_{\odot}$ , respectively [49, 50]. A long-term monitoring of GLS Q0957+561 revealed events of microlensing by bodies with a planetary mass of –  $10^{-5}M_{\odot}$  [51].

Thus, it is possible to obtain information about the nature of dark matter. For example, in the study of the ring-shaped GLS MG1654 + 561 [40], it was found that the radial distribution of the surface mass density is well described by the dependence  $\Sigma(r) \propto r^q$ . Comparison of the observed mass distribution with the empirical showed the presence of a hidden mass in the lens galaxy.

A lot of works have been devoted to the theoretical and observational study of microlensing [36,40,52,53,54,55]. Continuing this topic, let us say that the surface density of the masses in the direction of multiple images of the lensed quasar is of the order of the critical (see Eq. 1.14). Therefore, microlensing should occur quite often, and if the images are located sufficiently symmetrically, then continuously. This can be shown as follows. If we represent each microlens in the form of a small disk with a radius equal to the radius of its Einstein Ring, then the ratio of the area covered by such disks to the area of the selected section will correspond to the surface mass density expressed in units of critical density. This value is also called optical depth -k. At k = 1 Einstein's rings of microlenses overlap and cover the entire observable plane.





**Fig.1.7.** A distribution of the magnification factors in the source plane by a dense star field in a lensing galaxy. The parameters of microlenses were chosen in accordance with the model of the GLS Q2237 + 0305 [20].

**Fig.1.8.** The light curve of model corresponding to the light lines in Fig. 1.7. Solid and dashed lines - small and large angular size of the source. Each pair corresponds to three realizations of the random distribution of microlens positions [20].

Microlenses give a complex two-dimensional picture of the distribution of the magnification factors in the source plane, i.e. it consists of a set of caustics, which, as stated, correspond to an infinitely large gain. Calculation of the amplification of an arbitrary source during microlensing is a difficult task, the poet here uses numerical calculations of the lens equation, in particular, by the method of reverse ray tracing [36]. The method consists in calculating the path of the rays defined in a square grid in the opposite direction, i.e. from the observer to the plane of the source through the gravitational lens. When the ray system intersects the source plane, a gain distribution is obtained, which is proportional to the density of points. Due to the mutual transverse movements of the observer, GL and source, the apparent brightness of the lensed quasar can vary with time. In this case, it is assumed that the positions of the microlenses are fixed and the amount of amplification of the source relative to the calculated picture of the caustic. Examples of numerical calculations of the caustic pattern of the microlens system are shown in Fig. 1.7. The gray tone expresses the magnification as a function of the position of the quasar - the sequence of colors black-gray-white corresponds to an increase in the gain. The corresponding light curve is shown in Fig. 1.8. They are drawn in the case when the source moves along light, i.e. different implementations of microlens positions (Fig. 1.7). The figure shows that the larger the source size, the smoother the light curve.



**Fig.1.9.** The light curves of the four components of the GLQ Q2237 + 0305 according to the results of the OGLE monitoring program for 7 years [46,47].

From the point of view of observations and the study of microlensing in real GLS, the ideal object, among known objects, is Q2237 + 030. This system represents four images of the lensed quasar, which are located against a background of a relatively close spiral galaxy-lens. Due to the sufficiently high symmetry of the components location relative to the center of the lens, the delay time between the components amounts to hours or tens of hours. Therefore, it is assumed that any uncorrelated variations in the brightness of the components on longer time series are caused by the microlensing effect. Changes in the gloss of

the components of GLS Q2237 proceed almost continuously from the moment of its discovery. Figure 1.9 shows the light curves of all four components of this system over a nearly 6-year observation period by the OGLE program [46]. These changes are caused, in all probability, mainly by microlensing as the rays pass through the dense regions of the lens galaxy.

### §1.2. Problems of time delay

In this section, we want to highlight more widely the problems associated with the delay time ( $\Delta t$ ) in GLQ. The delay time is due to the fact that light from the source to each component comes in different ways (Fig. 1.10). And this means that any event that occurred in the source will first appear in the leading component, only after some time, equal to  $\Delta t$ , in the other component.



**Fig.1.10.** The figure illustrates the path difference between two refracted beams, which correspond to two components of an arbitrary GLS. The lens L, located between the source S and the observer O, forms two (S1 and S2) images of the source [20].

The analysis of observational data of the GLS makes it possible to obtain a number of important results concerning the nature of objects involved in lensing and the universe as a whole. Therefore, gravitational lenses, figuratively speaking, are cosmic telescopes. Due to the effect of amplification, we can observe the most distant objects of the universe, which in other cases it would not be possible to see - quasars. The most remote of the known quasars ( $z_q = 10.2$ ) was found precisely due to gravitational lensing in the cluster of galaxies [56]. Quasars, as is known,

are galaxies that are at an early stage of their evolution. This means that what is studied in the main theoretically can be confirmed by observations.

This can also be helped by the known value of the delay time. The fact is that because of the location of observatories it is impossible to observe continuously many quasars. The light curves suffer from seasonal breaks in observations. However, if the value of  $\Delta t$  is known for some GLS, then with its consideration it is possible to obtain a full-fledged quasar luminosity curve.

On the other hand, the GL allows directly, regardless of the intermediate distance indicators, to determine the Hubble constant H<sub>o</sub>. This method was proposed by S. Refsdal in 1964 [57,58] for the case when the source is a supernova lying behind and close to the line of sight of the lens-galaxy. The spread of the values of the Hubble constant, calculated from known GLQ, is approximately ~ 20%, from 52 to 69 with errors of the order of  $\pm$  20 days [53,54,59-63]. These differences are due to a number of factors: the uncertainty of the parameters in modeling the mass distribution in the lens galaxy, the influence of the unevenness of the light curves in determining  $\Delta t$ , etc. However, in general, we can assume that the obtained H<sub>o</sub> values are in good agreement.

In general, the delay time is related to the geometric and relativistic properties of the universe, so the function  $\Delta t$  is the sum of these two parts [20]:

$$\Delta t(\vec{\theta}, \vec{\beta}) = t_{geom} + t_{grav} = \frac{1 + z_d}{c} \frac{D_d D_s}{D_{ds}} \left( \frac{1}{2} \left( \vec{\theta} - \vec{\beta} \right)^2 - \psi(\vec{\theta}) \right) \quad (1.17)$$

where  $\psi(\vec{\theta})$  – two-dimensional potential GL.

As can be seen, the delay time is the main parameter of the GLQ, which makes it possible to judge the properties of the Universe and the structure of the lens (see, for example, [58,86]). Despite the fact that the  $\Delta t$  values for some systems (PKS1830-211, Q0957+561, B0218+357, B1600+435, B1422+231, B1608+656) were measured using light curves of radio [6,37,47,64,79] or X-ray

observations [28], yet the most significant part in determining the delay time is provided by optical monitoring.

Authors of the work [63] determined  $\Delta t$  between the components of the GLQ HE1104-1805. Shekhter et al. [62] obtained a delay time for two pairs of components of the GLQ PG1115+080. At the Scandinavian optical telescope, during recent years, some GLQ have been intensively observed. Several observation projects were carried out there and delay times were measured for a number of B1600+434, HE2149-2745, RXJ0911+0551, SBS150+530, FBQ0951+2635 [65,66,67]. The formal error value of these measurements is in the range from 5 to 25%, which corresponds to values from 4 to 24 days. [68] also reported the measurement of  $\Delta t$  between four components of the lensed quasar HE0435-1223.

Determining the delay time is a very difficult problem. A typical example is GLQ Q0957, to the measurements of which began immediately after its discovery. Different groups [44,55,69] who carried out observations both in the optical and in the radio bands gave results very different from each other (from 410 days to 540 days). This problem was resolved in [61,70], when a relatively sharp variability of the components in 1994-1996 was discovered with a difference  $\Delta t = 417 \pm 3$  days.

Nowadays time delay values are known for 29 GLQ only. In the next Table 1.1. we provide information about these objects.

<b>№</b> 1	<b>Lens name</b> O0142-100	<b>z</b> <sub>s</sub> 2.720	<b>z</b> <sub>d</sub> 0.490	<b>RA (J2000)</b> 01:45:16	<b>Dec (J2000)</b> -09:45:17	<b>m</b> <sub>s</sub> I=16.47/2	<b>m</b> <sub>d</sub> I=18.72	$\frac{N_{im}}{2}$	<b>size ('')</b> 2.24	dt (days) 95±10 (AB)	<b>Ref.</b> [234]
2	B0218+357	0.960	0.680	02:21:05	+35:56:13	I=19.28/2	I=20.06	2ER	0.34	$10.5\pm0.4$ (AB) -112.1 $\pm2.1$ (AB)	[261]
3	DES J0408-5354	2.375	0.597	-	-	-	-	4E	3.5	$-155.5\pm12.8 \text{ (AD)}$ $-42.4\pm17.6 \text{ (BD)}$ $-8.3\pm1 \text{ (AB)}$	[262]
4	HE0435-1223	1.689	0.460	04:38:14	-12:17:14	i=16.84/4	i=18.05	4	2.42	-1.1±1 (AC) -14.2±1 (AD)	[263]
5	SBS0909+523	1.380	0.830	09:13:01	+52:59:28	I=15.65/2	I=18.85	2	1.17	50.0±2 (AB)	[264]
6	RXJ0911+0551	2.800	0.770	09:11:27	+05:50:52	I=17.39/4	I=20.47	4	2.47	146.0±8.0 (AB)	[67]
7	FBQ0951+2635	1.240	0.260	09:51:22	+26:35:14	I=16.39/2	I=19.67	2	1.11	16.0±2.0 (AB)	[109]
8	Q0957+561	1.410	0.360	10:01:20	+55:53:49	I=15.99/2	I=17.12	2E	6.26	424.9±1.2 (AB)	[265]
9	SDSS J1001+5027	1.838	0.415	10:01:28	+50:27:56	-	-	2	2.86	119.3 ± 3.3 (AB) 40.6±2 (BA)	[266]
10	SDSS1004+4112	1.734	0.680	10:04:34	+41:12:42	i=17.53/4	i=18.42	4	15.99	821.6±2.1 (CA) > 1250 (AD)	[267]
11	SDSS1029+2623	2.197	0.550	10:29:13	+26:23:31	-	-	4	22.5	744.±10.0 (AB)	[268]
12	HE1104-1805	2.320	0.730	11:06:33	-18:21:24	I=16.17/2	I=20.01	2	3.19	-161±7.0 (AB) 4.4±2.0 (BA)	[269]
13	PG1115+080	1.720	0.310	11:18:17	+07:45:57	I=15.62/4	I=18.92	4	2.32	12.0±2 (AC) 16.4±2.5 (BC) 0.7±1.2 (AB)	[2]
14	RXJ1131-1231	0.658	0.295	11:31:51	-12:31:57	I=16.74/5	I=17.88	4	3.8	$1.1\pm1.5$ (AC) -90.6±2 (AD)	[270]
15	SDSS J1206+4332	1.789	0.748	12:06:29	+43:32:17	-	-	2	2.9	$111.3 \pm 3$ (AB)	[271]
16	SDSS J1339+1310	2.231	0.609	-	-	-	-	2	1.7	$47 \pm 5$ (AB) -17.4 $\pm 2.1$ (AB)	[272]
17	H1413+117	2.550	1.950	14:15:46	+11:29:41	I=16.44/4	H=18.61	4	1.35	-18.9±2.8 (AC) 28.8±0.7 (AD) 1 5+1 4 (AB)	[7]
18	B1422+231	3.620	0.340	14:24:38	+22:56:00	I=14.81/4	I=19.66	4E	1.68	$7.6\pm 2.5$ (AC) $8.2\pm 2.0$ (BC)	[201]

19	DSS J1515+1511	2.049	0.742	-	-	-	-	2	2	211±5 (AB)	[273]
20	BS1520+530	1.860	0.720	15:21:44	+52:54:48	I=17.61/2	I=20.16	2	1.59	129.0±3.0 (AB)	[46]
21	B1600+434	1.590	0.410	16:01:40	+43:16:47	I=20.87/2	I=20.78	2	1.4	51.0±4.0 (AB)	[53]
										31.5±3 (BA)	
22	B1608+656	1.390	0.630	16:09:13	+65:32:29	-	I=19.02	4	2.27	36.0±3 (BC)	[274]
										77.0±3 (BD)	
23	SDSS1650+4251	1.540	-	16:50:43	+42:51:45	I=16.98/2	I=20.5	2	1.23	49.5±1.9 (AB)	[275]
24	PKS1830-211	2.510	0.890	18:33:39	-21:03:39	I=22.27/2	I=21.42	2ER	0.99	26.0±4.0 (AB)	[276]
25	WEI 12033 1723	1 660	0.661	20.33.42	-17.73.13			4	25	35.5±1.4 (BA)	[277]
25	W11J20JJ-472J	1.000	0.001	20.33.42	47.23.43	-	-	4	2.5	62.6±3 (BC)	[211]
26	HE2149-2745	2.030	0.500	21:52:07	-27:31:50	I=16.29/2	I=19.56	2	1.7	103.0±12.0 (AB)	[65]
27	HS 2209+1914	1.070	-	22:11:30	+19:29:12	-	-	2	1.62	$20.0 \pm 5 (AB)$	[271]
28	SDSS 12222 12745	2 820	0.400					6	15.1	47.7±6 (AB)	[278]
20	5D55 J2227-2745	<b>35 3</b> 2222+27 <b>+</b> 3 2.020	2.820 0.490 -	-		_	-	- 0	15.1	722±24 (CA)	[270]
										0.25±2 (AB)	
29	Q2237+030	1.690	0.040	22:40:30	+03:21:28	I=15.16/4	I=14.15	4	1.78	1.46±2 (AC)	[154]
										0.085±2 (AD)	



**Fig.1.11.** Linear dependence between angular size of the GLQ and their time delay values.

Analysis of the table data showed the time delay values depend linearly on the angular sizes of the systems (fig.1.11). No other systematic patterns of time delay versus other system parameters, such as multiplicity or redshift, were found.

Difficulties in determining the exact value of the delay time is due, first, to the presence of seasonal interruptions in observations and a limited set of data, as well as the possible presence influence of microlenses. To take into account these effects, a number of methods have been developed that can be conditionally divided into discrete (using directly observational data) and those that use various kinds of analytical functions for data interpolation. Common to all methods is the following approach, namely, the light curve of a component with a constant step is shifted backward (forward), depending on whether it is driven or leading. For each shift, the degree of correspondence (e.g. correlation or minimization) between the two curves is calculated. In the end, the amount of shift corresponding to the maximum match is taken as the most probable value of the delay time.

One of the first methods of dispersed spectra was proposed [71,72]. With its help,  $\Delta t$  was calculated in the GLS Q0957, this method we used to determine the delay time in GLQ H1413 + 117 (see Chapter 4). Let us have two time series corresponding to the measurements of the brightness of the components A and B:

$$A_i = q(t_i) + \epsilon_A(t_i), \quad i = 1, \dots, N_A \tag{1.18}$$

and
$$B_j = q(t_j - \Delta t) + l(t_j) + \epsilon_B(t_j), \quad j = 1, \dots, N_B$$
(1.19)

where q(t) denotes the internal variability of a quasar, l(t) is a variability component that contains an unknown flux ratio (or stellar magnitude difference) and an additional low-frequency noise component due to possible microlensing,  $\Delta t$ is the delay time,  $\epsilon_A(t_i)$  and  $\epsilon_B(t_i)$  - errors in photometric measurements, which are used to determine statistical weights.

For each fixed combination  $\Delta t$ , l(t) a composite light curve C is generated, which includes all the values of A as they are and the magnitudes of component B, corrected for the delay  $\Delta t$  and the gloss shift l(t):

$$C_k(t_k) = \begin{cases} A_i, & \text{if } t_k = t_i \\ B_j - l(t_j), & \text{if } t_k = t_j + \Delta t \end{cases}$$
(1.20)

where k = 1, ..., N and  $N = N_A + N_B$ . Then the dispersion spectrum

$$D^{2}(\Delta t) = \min_{l(t)} D^{2}(\Delta t, l(t))$$
(1.21)

can be calculated analytically found through the deepest minimum with respect to  $\Delta t$ . Minimization (1.21) is written in the most general form and does not take into account a number of important factors, which we will describe below. In the first approximation, the statistical weights of photometric measurements are taken into account

$$D_1^2 = \min_{l(t)} \frac{\sum_{k=1}^{N-1} W_k (C_{k+1} - C_k)^2}{2\sum_{k=1}^{N-1} W_k}$$
(1.22)

where  $W_k$  statistical weights for photometric points in the composite light curve

$$W_k = W_{i,j} = \frac{W_i W_j}{W_i + W_j}.$$
 (1.23)

The next modification of (1.23) is expression

$$D_2^2 = \min_{l(t)} \frac{\sum_{k=1}^{N-1} W_k G_k (C_{k+1} - C_k)^2}{2\sum_{k=1}^{N-1} W_k G_k}$$
(1.24)

where  $G_k = 1$  if only  $C_{k+1}$  and  $C_k$  are taken from different components and  $G_k = 0$  in another cases. Thus, we avoid non-overlapping areas on the light curve. Of course, (1.24) will give more plausible results, since in case (1.22) we can deal with the dominance of only one of the components.

The pairs of consecutive observations are included in the variance estimate  $D_1$  and  $D_2$  without taking into account the distance between two time instants  $t_i$  and  $t_j$ . Such gaps can sometimes reach significant values. Most often this is caused by breaks between the seasons, or because of poor conditions for observation. Thus, two observations, one of which occurs before such a long period of time, and the other after this interval, can make an erroneous contribution to the overall variance estimate. Long-term correlation over long intervals is, of course, excluded in the light curves of quasars. Therefore, in order to take into account such a state, an additional constraint in estimating variance is added:

$$D_3^2 = \min_{l(t)} \frac{\sum_{k=1}^{N-1} S_k W_k G_k (C_{k+1} - C_k)^2}{2 \sum_{k=1}^{N-1} S_k W_k G_k}$$
(1.23)

where

$$S_{k} = \begin{cases} 1, & \text{если } |t_{k+1} - t_{k}| \leq \delta, \\ 0, & \text{если } |t_{k+1} - t_{k}| > \delta, \end{cases}$$

 $\delta$  there is a maximum distance in time between two observations, which can still be regarded as nearby.

Other methods were based on the use of a discrete cross-correlation function (DCF) [73] in the form:

$$DCF(t) = \frac{1}{N} \sum_{i,j} \frac{(a_i - \bar{a}) \cdot (b_i - b)}{\sqrt{(\sigma_a^2 - \varepsilon_a^2) \cdot (\sigma_b^2 - \varepsilon_b^2)}}$$
(1.24)

where N – number of pair data  $(a_i, b_i)$  taking into account the bias t,  $\varepsilon_x$  – measurement errors,  $\sigma_x$  – standard deviation and  $\bar{a}, \bar{b}$  – the average value of the corresponding components. Various authors used modified versions of the function (1.24), for example, a locally normalized *DCF* [69], continuously determined by *DCF* [74], a combination of the last two [75,76]. As a measure of the

correspondence between the data, one can also use the minimization of  $\chi^2$ , for example in the following form:

**N** 7

$$\chi^{2} = \frac{1}{N-2} \sum_{i=1}^{N} \frac{(A(t_{i}) - B(t_{i} + \Delta t) + \Delta m)^{2}}{\sigma_{A}^{2} + \sigma_{B}^{2}}$$
(1.25)

here N – also number of pair data,  $\Delta m$  – the weighted average difference between the two data of the two components. In this way, in [46] we succeeded in obtaining the most probable value of the delay time  $\Delta t = 130 \pm 3$  days in SBS1520+530.

Analysis of discrete methods has shown that they work well in the case when there is a lot of data corresponding to a long period of observations, and the intervals between the seasons should be less or of the order of the expected value of  $\Delta t$ . Otherwise, especially when there is little data, it makes sense to interpolate the data and thus obtain full-fledged light curves. Interpolation can be carried out in various ways, for example, by Fourier series expansion, linear interpolation, approximations by polynomials of different degrees, and other functions. However, the main drawback of this approach is that it is necessary to make some assumptions, for example, about the slowly changing variability or the absence of microlensing, etc. At the same time, one can get false information about variability or, on the contrary, miss some features. This method should be used with great care.

# §1.3. Observational aspects of the CCD photometry of GLQ and the choice of objects

The main goal of this work is to carry out CCD observations and photometric processing of the selected GLQ at the Maidanak Observatory. These results will help us in the following tasks: development of the method of photometry of point sources in the case of GLQ, determination of time delays and its depending on other system parameters, resolving of a microlensing problems, calculation of the Hubble constant, etc. Carrying out successful observation of GLQ first of all we need to choose the most promising of them. This is necessary in order to ensure the reception of high-quality images for a long time. When selecting objects for observation, a number of parameters had to be taken into account-the position on the celestial sphere, the relative brightness, the distance between the components, the sensitivity of the light detector and the optical system of the telescope as a whole. At the beginning of our work (spring 2008), the number of GLQ was about 100 (currently they are about 150-200). It is impossible to observe all of them. Therefore, when selecting the objects of observation, we adhered to the following important criteria:

1. Conditions of visibility. By declination in the equatorial coordinate system, we were limited to  $-10^{\circ} < \delta < +70^{\circ}$ .

The angular distance between the components should not be less than
 1" of the arc.

3. The visible magnitude of the observed objects should preferably be no more than  $19 \div 20$  mag.

According to these criteria, we selected about 20 GLQ that could be observed in the Maydanak Observatory, such as UM 673, SBS0909+523, RXJ0921+4529, FBQ0951+2635, Q0957+561, PG1115+080, H1413+117, B1422+231, SBS1520+530, Q2237+030, et al. General information about these objects is presented in the database [77].

To study and obtain reliable information about GLS is possible only by continuous long-period observations of the investigated objects. As a rule, images of GLQ are relatively dim turned objects, therefore, for successful monitoring, at least three conditions are required: a) an observatory with the best astroclimatic indicators; b) a telescope with a sufficiently large angular resolution and diameter of the main mirror; c) a high-quality photosensitive device. These conditions are fully satisfied with the Maidanak Astrophysical Observatory (MAO) and its main 1.5-m. telescope AZT-22.

According to its astroclimatic characteristics, MAO is considered one of the best observatories not only within the CIS (Commonwealth of Independent States), but also in the world. It is located at an altitude of 2600 meters above sea level in the Kashkadarya region of Uzbekistan. The pioneer observations of GL in the MAO began in the early 90s of XX [78]. Our work is a natural continuation of previously performed observations of individual GLQ, which proved that GLQ monitoring can be successfully carried out in MAO using the AZT-22 telescope [79,80]. The results of long-term measurements using a specialized DIMM device [81] showed that an average image quality (FWHM) at the zenith is ~0.70", and the total time of a clear photometric sky is approximately 2000 hours per year (this is 60% of general dark time).

Telescope AZT-22 uses in maximum the unique capabilities of the observatory - it is located on a secluded summit and has a high enough tower (13.5 meters to the point of intersection of the main axes). This contributes to minimizing the impact of the surface atmospheric layer, where heterogeneities are constantly present due to changes in soil temperature, which is very strongly felt in the summer months. In addition, the telescope tower is equipped with a ventilation system through a special air vent, which ensures a quick equalization of the temperature of the dome space with the surrounding air.

The telescope AZT-22, with a diameter of the main mirror of 1.5 meters, has the Ritchie-Chretien optical system. Replaceable secondary mirrors can be realized as a wide-angle system with a luminosity of 1:7.7, and a system that creates images with a large scale (luminosity 1:17). According to the results of factory tests, the quality of AZT-22 optics is close to diffraction limited. The residual root-mean-square error of the wave front is of the order of  $0.15\lambda$  [82].

The analysis of the quality of the images obtained earlier and the assessment of the influence of astroclimatic conditions on them during real observations showed that in the winter-spring season the average FWHM value is  $0.8 \div 0.9$  arc seconds, reaching  $0.45'' \div 0.55''$  on the best images [21]. In the case of well-

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established weather, image quality tends to gradually improve and stabilize at around 23h local time. It should also be noted that there is a clear correlation between the image quality estimates of the AZT-22 received at the telescope and simultaneously on the DIMM complex about 100 meters south of AZT-22 [34].

To obtain high-quality images, a good photosensitive device is also necessary. A high quality CCD-camera BroCam (SITe ST-005A) has been installed on the AZT-22 telescope since 2000-2006. The complete set consists of a CCD camera with a cryostat, power supplies, a controller, control computers and a portable fore-vacuum pump. In addition, there is a system for supplying liquid nitrogen to the chamber and a special optical attachment for testing the camera.

In addition, this chamber is equipped with a set of UBVRI glass filters closely corresponding to the Johnson-Kuzin photometric system [83]. Calculations of the relative spectral sensitivity of the filters showed that the highest sensitivity in the BroCam receiver is achieved in the VRI bands, and the smallest in the U band. The normalized curves with respect to the standard bands of the Johnson-Coinsins UBVRI show that the V and R filters are most consistent with the standards, but the filters U, B and I differ [21]. Therefore, we carried out observations mainly in the filters V and R. Later we used new CCD-camera SNUCam, which parameters are very similar to the previous one.

### §1.4. Cosmological application of the gravitational lensing: determination of Hubble constant

Gravitationally lensed quasars are known to potentially provide estimates of the Hubble constant  $H_0$  from measurements of the time delays between the quasar intrinsic brightness variations seen in different quasar images [57,58]. Since a phenomenon of gravitational lensing is controlled by the surface density of the total matter (dark plus luminous), it provides a unique possibility both to determine the value of  $H_0$  and to probe the dark matter content in lensing galaxies and along the light paths in the medium between the quasar and observer. By now the time delays have been measured in more than 20 gravitationally lensed quasars resulting in the values of H0 that are generally noticeably less than the most recent estimate of H0 obtained in the Hubble Space Telescope (HST) Hubble Constant Key Project with the use of Cepheids -  $H_0 = 72 \pm$  $8 \, km \, s^{-1} Mpc^{-1}$  [185]. This discrepancy is large enough and, if the Hubble constant is really a universal constant, needs to be explained. A detailed analysis of the problem of divergent  $H_0$  estimates inherent in the time delay method, and the ways to solve it can be found in e.g. [89,148,157,164,165,166], and in many other works.

The main sources of uncertainties in determining  $H_0$  are

- i. low accuracy of the time delay estimates caused by poorly sampled and insufficiently accurate light curves of quasar components, as well as by microlensing events and, as a rule, by low amplitudes of the quasar intrinsic variability;
- ii. difference in the values of cosmological constants adopted in deriving  $H_0$ ;
- iii. invalid models of mass distribution in lensing galaxies.

The way to reduce the effect of the first source of errors is clear enough: more accurate and better sampled light curves of a sufficient duration are needed. A choice of the cosmological model is usually just indicated – this is mostly a question of agreement. As to the third item, here the problem of estimating the Hubble constant encounters the problem of the dark matter abundance in lensing galaxies.

The problem of determining the Hubble constant from the time delay lenses is known to suffer from the so-called central concentration degeneracy, which means that, given the measured time delay values, the estimates of the Hubble constant turn out to be strongly model dependent. In particular, models with more centrally concentrated mass distribution (lower dark matter content) provide higher values of  $H_0$ , more consistent with the results of the local  $H_0$  measurements than those with lower mass concentration towards the centre (more dark matter). Moreover, it

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has long been noticed that the time delays are sensitive not only to the total radial mass profiles of lensing galaxies, but also to the small perturbations in the lensing potential (e.g. [90,91,92]. It is interesting to note that this effect has been recently proposed as a new approach to detect dark matter substructures in lensing galaxies [170].

The Hubble constant–central concentration degeneracy is a part of the well known total problem of lensing degeneracies: all the lensing observables, even if they were determined with zero errors, are consistent with a variety of the mass distribution laws in lensing galaxies. A strategy for solving this non-uniqueness problem could be a search through a family of lens models that are capable of reproducing the lensing observables [92,93]. Then many models can be run in order to infer a probability density for a parameter under investigation, e.g. for  $H_0$  [93]. In the most recent studies the authors of [89,94] have shown that, in such an approach, the discrepancy between the  $H_0$  value determined from lensing and with other methods can be substantially reduced if non-parametric models for mass reconstruction are used, which can provide much broader range of models as compared to the parametric ones.

In defining priors on the allowed space of lens models, it is naturally to assume that lensing galaxies in the time delay lenses are similar in their mass profiles to other early-type ellipticals that are presently believed to be close to isothermal and admit the presence of the cold dark matter haloes. The isothermal models are also consistent with stellar dynamics, as well as with the effects of strong and weak lensing.

The quadruply lensed quasars are known to be more promising for solving these problems as compared to the two-image lenses since they provide more observational constraints to fit the lens model. 10 astrometric constraints can be presently regarded as measured accurately enough for most systems. This especially concerns the relative coordinates of quasar images. As to the lensing galaxy, its less accurate coordinates are often the only reliable information about

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the lensing object known from observations, with other important characteristics being derived indirectly. This situation is inherent, e.g. in PG 1115+080 with its faint, 0.31 redshift galaxy. Of other observational constraints, the time delays and their ratios are very important. In quadruple lenses, the time delay between one of the image pairs is usually used to determine H0, while the other ones form the H0 independent time delay ratios to constrain the lens model [148].

It has long been known that the observed positions of multiple quasar macroimages are well predicted by smooth regular models of mass distribution in lensing galaxies, while their brightness ratios are reproduced by such models poorly (e.g. [95,96,174]. The first systematic analysis of this problem called 'flux ratio anomalies' was made in [174], who assumed that the anomalies of mutual fluxes of the components in some lenses can be explained by the presence of small-scale structures (substructures) in lensing galaxies or somewhere near the line of sight.

A popular model of forming hierarchical structures in the Universe with a dominant content of dark matter is currently known to poorly explain the observed distribution of matter at small scales. In particular, the expected number of satellite galaxies with masses of the order of  $M_G \approx 10^8 M_{\odot}$  remained after the process of hierarchical formation is completed is an order of magnitude larger than a number of dwarf galaxies with such masses actually observed within the Local Group (see [97,98]. One of the solutions of this contradiction is a suggestion that some substructures, especially those with low masses, are not luminous.

Authors of [99] were the first to note that the dark matter paradigm can naturally explain existence of substructures in galaxies lensing the remote quasars, as proposed in [156] to interpret the anomalies of mutual fluxes of quasar macroimages, and vice versa, confirmation of substructures with masses from  $10^6 M_{\odot}$  to  $10^8 M_{\odot}$  is capable of removing the contradiction between the predicted number of the low-mass satellite galaxies and that one actually observed. The idea turned out to be intriguing and was immediately taken up [100,101,102, 159,202]. Investigation of flux ratio anomalies in gravitationally lensed quasars is presently believed to be a powerful tool in solving the problem of the dark matter abundance in the Universe. It is intensively discussed in numerous publications [e.g. 103, 104,162,163,175,176,177, 179,180,182].

#### §1.5. Problems of image processing methods of GLQ

As a result of reading the information from the CCD camera, we obtain a two-dimensional matrix of numbers (frame) whose element numbers correspond to the numbers of the rows and columns of the photosensitive chip, and the intensity values in this pixel. The registration of the number of photoelectrons is described by the Poisson distribution [84]:

$$p(n,t) = \frac{(\overline{N} \cdot t)^n}{n!} e^{-\overline{N}t}$$
(1.26)

where n – the number of photoelectrons generated per unit time, p(n, t) – the probability of appearance of electrons during time t,  $\overline{N}$  – dispersion. Therefore, the root mean square fluctuation of the recorded photons is

$$\sigma(N) \cong \sqrt{\overline{N}} . \tag{1.27}$$

This value in stellar photometry is also called the *signal-to-noise ratio* (SNR), expressed as

$$SNR = \frac{signal}{noise} = \frac{\overline{N}}{\sqrt{\overline{N}}} = \sqrt{\overline{N}}$$
 (1.28)

In this case, the inverse *SNR* is the value of the measurement error  $\overline{N}$ . But in reality, an instrumental signatures of CCD-detectors and unwanted signals are added to the useful signal that lead to an increase in the noise level. Usually we deal with the following factors and unwanted signals:

- the dark current – even in the absence of illumination, CCDs produce a residual or 'dark' current which depends on the detector temperature and exposition time. Detectors we used had high quality cooling systems, which minimized noise of this effect; - zero offset caused by the presence of a constant output voltage;

readout noise – this is electronic amplifier noise, usually quoted in electrons.
 Does not depend on exposure time;

- the sampling noise that arises due to rounding in the conversion of the analog code;

- flat curvature caused by unevenness of the sensitivity (quantum efficiency) over the CCD-chip which changes from pixel to pixel;

- sky noise - the night sky is not dark. Sky background increases at longer and longer wavelengths;

- cosmic rays, is a result of bombardment of the CCD-chip by high-energy particles. This can be both particles of cosmic radiation, and their local sources. This type of noise becomes significant in frames of sufficiently long exposures.

In general, the processing of frames received on CCD cameras consists of two stages - primary processing and photometry. The methods of primary processing are quite standard [85] and aims of this step are:

- removal of instrumental signatures, like dark current and field curvature
- masking of unwanted signals, like cosmic rays, stellar halos and satellite tracks
- photometric and astrometric calibration
- coaddition of individual frames

The offset value of zero and dark current was taken as the average value of the intensity in the overscan region and was subtracted from the intensity of each pixel in raw frames. To account for the uneven sensitivity superflats were used – an averaged images of a flat sky. They were obtained during the morning and evening twilight in all filters on average 5 frames. Then they were averaged and normalized by average flux value. We should say today there are a number of universal software packages, such as IRAF, MIDAS, AIPS, IDL, etc. These packages satisfy almost all the requirements for image processing. In practice, the *ccdproc* task in the IRAF package was used for pre-processing.

After the frames are pre-processed, it is required to carry out photometric processing of the lensed components of our GLQs. At the present time, there are many photometric methods, here we describe the aperture and PSF photometry used by us for double quasars.

The aperture method is the simplest of all which summarizes the intensity of the pixels within a certain area around the study object. Usually this area is a circle of a certain radius (for photometry of stars), but it can be a ring or an ellipse. At the same time, instrumental shine is calculated using a simple formula:

$$m_{intr} = -2.5 \cdot \log\left(I_{S+F} - \left(\frac{N_{S+F}}{N_F}\right) \cdot I_F\right)$$
(1.29)

where  $I_{S+F}$  – the sum of the counts from the star along with the background,  $N_{S+F}$  – the number of pixels within a given area where the count is made, where the count is made  $I_{S+F}$ ,  $I_F$  – total background brightness,  $N_F$  – area where the brightness of the background was calculated. The size of the aperture is chosen so as to most accurately measure the brightness of the star, but at the same time to minimize the influence of neighboring objects. This is usually equal to several FWHMs. In practice, this method is implemented in the *phot* program from the *IRAF* package. In this case, you just need to specify the coordinates of the stars whose brilliance should be measured and the radius of the aperture. The output is a text file with stellar magnitudes and a measurement error.

The aperture method is convenient for single or isolated objects, which do not interfere with neighboring objects. However, the components of images of almost all GLQ are very complex configurations, especially since the distances between them are of the order of 2-3 FWHM. Generally speaking, photometry of each GLQ requires a individual approach. The main difficulties in photometric observations and data processing arise from the following factors, which require special approaches to this problem [86]:

- the majority of GLQ are manifested as weak point or blurred objects;

- components of the lensed source often located on the background of lensing galaxies, which create background gradient regions. Because of this, there are errors in estimating the value of the local background near the individual components;
- the components of the GLQ are very close to each other and often overlap (a case of dense fields). As a result, it is difficult to measure the intrinsic brightness of their components;
- the presence of a diffraction rays and a halo from close bright stars. The difficulties specified in paragraphs 2 and 3 arise;
- remoteness or uncertainty of the comparison star, optical problems of the telescope, etc.

Therefore, more complex methods and programs are used for flux measure of GLQ's lensed images. One of methods is the PSF fitting. Before describing the essence of this method, let us dwell briefly on the mechanism of image formation on a CCD detector.

The point is that the radiation coming from the object, passing through the atmosphere and the optical system of the telescope, is essentially distorted. This is influenced, on the one hand, by variability of the optical properties of the atmosphere, and on the other hand by optical system of the telescope. As a result, the image is "blurred". Image of the object is the sum of fluxes from images of all points constructed by the optical system. Thus, it is possible to express the relationship between the observed  $I_{obs}$  and true image  $I_0$  of the object as following

$$I_{obs}(r) = \int_{-\infty}^{+\infty} S(r - r') \cdot I_0(r') \, dr'$$
(1.30)

where S(r - r') - the function characterizing distortion of the system (by the system we mean the telescope and the atmosphere). In other words, this is the flux distribution in the image of a point source located at the origin of coordinates and having a unitary integral intensity

$$\int S(r - r') \, dr' = 1 \,. \tag{1.31}$$

This function called point spread function - PSF. To describe it various distributions can be used, for example, Gauss  $G(r) \sim \exp(-r^2/(2\sigma^2))$ , Lorenz  $L(r) = (1 + (r^2/\sigma^2)^{\beta})^{-1}$ , Moffat  $M(r) = (1 + r^2/\sigma^2)^{-\beta}$ , where  $\sigma$  – full width at half maximum (FWHM),  $\beta$  – some coefficient.

The PSF photometry method is based on the assumption the shape of the stellar image profile is unchanged over CCD-chip area irrespective of brightness, only possible changes in the PSF are taken into account, depending on the position in the image. It follows from eq. (1.30) the flux distribution of point source can be estimated by comparing the normalizing factors for a PSF on the same frame.

The problem of photometry of the GLQ's components can be considered as a special case of photometry in dense stellar fields [126]. This method is very well implemented in programs like *Daophot*, *Galfit*, etc. In these programs, the flux distribution in the PSF and coordinates of the GLQ's components is assumed to be known, and we calculate flaxes of them. By the term fitting is understood as finding the best correspondence of the given PSF and the real distribution of brightness in the GLQ images. Moreover, the profile of a star can be represented in the form of an analytic function, either in the form of numerical functions, or a combination of both.



a)



**Fig. 1.12.** The results of subtraction from the star image according to analytical (a) and numerical (b) PSF [21].

б)

The degree of correspondence of a real-time flux distribution calculated in some way by the PSF can be judged by the residuals of the subtraction of the PSF from the real image. In Fig. 1.12 the results of subtracting the analytical (a) and numerical PSF (b) from the star image are shown. It can be seen that the numerical PSF is much more adequate to reproduce the real intensity distribution in the point source.

Therefore, the use of combined PSF is considered more correct, as was done in [87,88]. In this case, the PSF consists of two parts: an analytic function in the form of a two-dimensional Gaussian:

$$G(x,y) = F_0 \exp\left\{-\frac{1}{2}\left[\frac{(x-x_0)^2}{\sigma_1^2} + \frac{(y-y_0)^2}{\sigma_2^2}\right]\right\}$$
(1.32)

and the remainder of the subtraction of this function from the image of the star. In this case, the subtraction residue complements the analytic function and interpolation errors are obtained much less.

On this principle, the Daophot program is developed, which uses the model of the point-scattering function, in which an attempt is made to combine analytical and empirical approaches. The profile of the stars is approximated by some analytic function, and then the averaged two-dimensional residue table is calculated, which takes into account the difference of the real profiles from the selected analytical representation (mainly in the wings region). A circular region of a certain radius is chosen around each of the PSF stars, to which the image of the star must fully fit (to a level where the noise from the background is comparable with the contribution from the wings of the profile of the star) -the so-called writing area. Using this area, instead of analyzing the entire frame as a whole, allows you to significantly speed up the process of analyzing the profiles of stars by reducing the counting points. The profile of the stars is approximated by one of the selected analytic functions (Gauss, Lorenz, Moffat) or a combination of them, by solving *least squares* method of the nonlinear system of equations (1.32). A detailed description of photometric methods applied to specific objects, we give in the relevant chapters.

# CHAPTER II. MICROLENSING VARIABILITY IN DOUBLE GLQs: SHORT-TIME-SCALE EVENTS OR A LONG-TIME-SCALE FLUCTUATION?

#### §2.1. Observations of FBQ 0951+2635

FBQ 0951+2635 was discovered three decades ago [105]. This is a double quasar (consisting of two components A and B) at redshift  $z_s = 1.246$ , which is gravitationally lensed by an early-type galaxy at  $z_l = 0.260$  [106]. The main lensing galaxy probably belongs to a group of galaxies at similar redshift [107].

Optical follow-up of FBQ 0951+2635 has been done in the current decade. This includes early imaging and monitoring with the Hubble Space Telescope (HST; [108]) and the Nordic Optical Telescope (NOT; [109]), as well as imaging from the Sloan Digital Sky Survey (SDSS; [110]). The 2.5-yr monitoring campaign at the NOT focused on the R pass-band. These R-band frames allowed [109] and [111] to study the time delay between components and early extrinsic variability. The time delay between A and B is of about 2 weeks [109], and there is clear evidence for R-band extrinsic variations during the 1999–2001 period [111]. Also [111] showed a possible gradient of about 0.1 mmag d<sup>-1</sup>, as well as a possible 50-mmag event on a time-scale of several months. These extrinsic fluctuations (which are not originated in the source quasar) were attributed to microlensing by collapsed objects within the lensing galaxy (e.g. [112], and references therein).

However, the information obtained in the first years of monitoring with the NOT (1999–2001) does not permit to decide on the true microlensing variability, so additional imaging is required to address this issue. For example, in fig. 2 (middle and bottom panels) of [111], one sees that the microlensing gradients can account for basically all variability without the need of introducing additional short-term (~months) microlensing events (see the distributions of points around the linear fits). Alternatively, in the same panels of fig. 2 of [111], sets of short-term microlensing events can also explain the observed variations. Thus, it is

unclear what kind of microlensing fluctuations occur in FBQ 0951+2635: shorttime-scale events or gradients lasting years (tracing a long-time-scale event)? Although both kinds of fluctuations may be present, one reasonably expects the presence of a dominant kind accounting for most of the observed microlensing variability. In this Chapter, we try to identify the dominant microlensing variations in double lensed quasars.

#### §2.2. FBQ 0951+2635: Light curve and flux ratio in the R band

FBQ 0951+2635 is part of a compact lens system, since the two quasar components are separated by 1.1 arcsec [105] and the very faint lensing galaxy is 0.2 arcsec away from the faintest component B [109].



**Fig. 2.1.** Central region of a Maidanak R-band frame of FBQ 0951+2635. This frame was taken under very good seeing conditions (FWHM = 0.75 arcsec) on 2004 January 22. The exposure time was 180 s, and we show the logarithm of counts in each pixel.

Fortunately, the lensing galaxy remains too faint to be detected with a standard R filter [109], and this is an advantage while doing photometry. Thus, the system can be described by two stellar-like objects. There are two field stars near the lensed quasar: the bright star S1 and the faint star S3 (see Fig. 2.1). The full width at half-maximum (FWHM) of the seeing disc is measured on the S1 star, which is also used to estimate the point spread function (PSF) of the stellar-like sources. This estimation allows us to obtain PSF-fitting photometry for the double quasar.

A single-epoch magnitude difference  $(m_B - m_A)$  could not represent the magnitude difference at the same emission time  $(\Delta m_{BA})$ , because one should take into account the 16-d time delay between components [109]. However, the typical variability of FBQ 0951+2635 on a time-scale of ~2 weeks gives us the typical amplitude of the deviations  $\delta = m_B - m_A - \Delta m_{BA}$ , so a correction for simultaneity can be estimated from variability studies. Throughout this paper, we derive  $\Delta m_{BA}$  values from single-epoch magnitude differences, whose photometric uncertainties are properly enlarged to incorporate the simultaneity error  $\sigma_{sim}$  (i.e. the typical amplitude of  $\delta$ ). We adopt  $\sigma_{sim} = 0.03$  mag, which is consistent with the *R*-band root-mean-square (rms) variability of FBQ 0951+2635 over a 16-d period (see details below), as well as with the *i*-band variability over such time-scale (see Section §2.2.). All single-epoch measurements of  $\Delta m_{BA}$  include the 0.03-mag uncertainty added in quadrature to the photometric errors. The flux ratio between components is given by  $A/B = 10^{0.4 \times \Delta m_{BA}}$ .

The Maidanak gravitational lens monitoring programme is being conducted by an international collaboration of astronomers from Russia, Ukraine, Uzbekistan and other countries. The median FWHM (~0.7 arcsec) and number of clear nights (~200 nights per year) at Mt. Maidanak [81,113] permit to obtain high-resolution images of compact lens systems (e.g. [114]), and here we present and analyse Rband observations of FBQ0951+2635. These Maidanak homogeneous observations from 2001 April to 2006 May are an important tool to understand the microlensing variability in the double quasar.

Our Maidanak monitoring consisted of 190 frames (exposures) in the R band, which were taken with the 1.5-m AZT-22 Telescope at Mt. Maidanak on 37 different nights (see Table 2.1). We used the LN2-cooled CCD camera (BroCam) with SITe ST-00A CCD chip. This 2000 × 800 CCD detector has pixels with a physical size of 15  $\mu$ m, giving angular scales of 0.135 arcsec/pixel (long-focus mode) and 0.268 arcsec/pixel (short-focus mode). The gain and readout noise are 1.2 e<sup>-</sup>ADU<sup>-1</sup> and 5.3 e<sup>-</sup>, respectively.

The pre-processing of each frame consists of bias subtraction, overscan trimming, flat-fielding, and cosmic rays cleaning. For each observation night, we have two or more individual frames obtained under good seeing conditions:  $\langle FWHM \rangle \sim 1$  arcsec. For example, in Fig. 2.1 we display the central region (1 × 1 arcmin<sup>2</sup>) of an 180-s exposure in subarcsecond seeing conditions. This R-band image (logarithm of counts over the FBQ 0951+2635 field) includes the S1 star (~16.6 mag), the two components A and B (~17.1 and ~18.3 mag, respectively) and the S3 star (~19.5 mag). Moreover, the average signal-to-noise ratio (S/R; aperture radius of 2 arcsec, i.e. 2 × FWHM) of an 18 mag star is  $\langle S/R \rangle \sim 50$ . Thus, most of the nightly FWHM values are less than the separation of the double quasar, and there is significant signal associated with the faintest quasar component.

Table 2.1. Optical	observations of FBQ	0951+2635
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Observatory	Telescope	Camera	Filter	Period	Nights	Exposures/night
Maidanak	1.5-m	Bro-	R	2001 April	1	2 (120+180 s)
(Uzbekistan)	AZT-22	Cam		2002 March	3	2-4 (180/240 s)
				2003 Apr-May	2	2 (180 s)
				2004 Jan-May	14	3-5 (180/210 s)
				2004 Dec-	6	4-8 (180 s)
				2005 Apr		
				2005 Nov-Dec	3	2 (300 s)
				2006 Apr-May	8	10 (180 s)
La Palma	2-m	RAT-	i	2007 Feb-May	52	5 (100 s)
(Canary Islands)	Liverpool	Cam		-		

PSF-fitting photometry leads to instrumental fluxes of components and field stars. These fluxes are calculated using the instrumental photometry pipeline presented in [115]. We also obtain calibrated light curves (in mag) of A, B and S3, as well as single-epoch magnitude differences  $m_{\rm B} - m_{\rm A}$ . The individual values are then combined to form nightly means and standard deviations of means (i.e. standard errors). The standard errors of  $m_{\rm B} - m_{\rm A}$  are properly enlarged because we are interested in magnitude differences at the same emission time (see explanation above). The fluxes of A, B and S3 on 2001 April 15 are also compared to the corresponding NOT fluxes at close dates [109]. We do not find offsets in the fluxes of A and S3, but the flux of B is slightly higher (60 mmag) than the NOT level around 2001 April 15. This offset is very probably due to a small contamination by the lens galaxy light (which is absent in the NOT fluxes), so a 60-mmag correction is taken into account when obtaining final fluxes of B. Each of the four combined curves ( $m_A$ ,  $m_B$ ,  $m_{S3}$  and  $\Delta m_{BA}$ ) incorporates 37 data points. These data points and other quantities of interest can be found in the Table 2. of the paper [2].

Although the standard errors may be good estimators of the nightly photometric errors in  $m_A$ ,  $m_B$  and  $m_{S3}$  changes in the colour coefficient and/or the possible inhomogeneous response over the camera area could produce a substantial amount of additional noise throughout the several years of monitoring (e.g. [115]). We check this possibility by comparing the average standard error of  $m_{S3}$  and the standard deviation of  $m_{S3}$  over the whole monitoring period. There is a bias factor of about  $\sqrt{3}$ , so we multiply the average standard errors of  $m_A$ ,  $m_B$  and  $m_{S3}$  by  $\sqrt{3}$ to derive typical photometric errors. These final uncertainties are 9 mmag (A), 24 mmag (B) and 28 mmag (S3). The top panel in Fig. 2.2 shows the final brightness records of the double quasar and the S3 star. Filled circles, filled triangles and open circles trace the behaviours of A, B (shifted by -0.7 mag) and S3 (shifted by -1.2mag), respectively. The almost parallel fading by ~0.6 mag of the two components indicates the presence of long-time-scale intrinsic variability (extrinsic variations are discussed below). Around day 2750 (JD 245 1100), one can see an important scatter in the light curves of A and B. This scatter was produced when the camera (BroCam) reached the end of its lifetime.

In general, sampling is unsuitable when doing analysis of short-time-scale (intra-year) variability. However, this kind of study was possible in 2004 (see Table 2.1), i.e. around day 2000 in Fig. 2.2 (top panel). In the bottom panel of Fig. 2.2, while the filled circles describe the time evolution of the flux of A, the filled triangles represent the time- and magnitude-shifted light curve of B.

The fluxes of B are advanced in 16 d (time delay by [109]) and increased by 1.246 mag (average magnitude difference between the time-shifted record of B and the light curve of A; although bins with semi-size of 4 d are used, the average magnitude difference is within the 1.24–1.25 mag interval for any bin semi-size below 6 d). We show that there is good agreement between both brightness records, i.e. the two records are consistent with each other in the overlap periods. For this reason, we can state that short-time-scale microlensing is absent or elusive. The time-delay-corrected flux ratio (R-band data from day 1900 to 2050) is  $A/B = 3.15 \pm 0.05$  (1 $\sigma$ ).



**Fig. 2.2.** Maidanak R-band light curves of FBQ 0951+2635. Left: A (filled circles), B - 0.7 mag (filled triangles), and S3 - 1.2 mag (open circles). Right: comparison between the A fluxes in the first semester of 2004 (filled circles) and the time- and magnitude-shifted brightness record of B in that semester (filled triangles). See main text for details.

Taking the error bars into account, the true variation in the flux of B could be of about 30 mmag. This is in good agreement with the adopted variability over a 16-d period. Moreover, the  $\sim 0.1$  mag jump (considering central values) is only defined by two consecutive fluxes of the B component. In other words, it is a very

poorly sampled variation in the faintest and noisest component, which is most likely caused by observational noise.

The 37 single-epoch measurements of  $\Delta m_{BA}$  are grouped in seven different time intervals. We compute the weighted average and its uncertainty for each interval. These averages are then translated into seven *R*-band flux ratio measurements with an accuracy of 1–3 %. The new flux ratio values are depicted in Fig. 2.3 (filled circles). For example, the flux ratio around day 2000 (3.17 ± 0.03; 1 $\sigma$ ) is consistent with the result in two paragraphs above, which takes the time delay correction into account.

Around day 2750, we obtain two estimates of A/B because the photometric data show a large scatter (see above). First, we use all available nights (last open circle in Fig. 2.3). Secondly, only high-quality nights, i.e. FWHM < 1.1 arcsec and S/R > 50, are considered (filled circle above the last open circle in Fig. 2.3). This second estimation seems more reliable, so the first one is assumed to be biased. We also compare the NOT flux ratio at an intermediate epoch and the corresponding Maidanak ratio from the former ST-7 CCD camera (which was operating in that epoch). The Maidanak/ST-7 1 $\sigma$  measurement, 2.58 ± 0.05 (first open circle at day 484), is in excellent agreement with the NOT gradient and limits (dashed lines; see later in this paragraph).

A few complementary frames are used to check the reliability of the A/B values from the Maidanak/BroCam homogeneous monitoring. The SDSS archive [110] contains a frame of FBQ 0951+2635 in the *r* band. This frame was taken under very good seeing conditions (FWHM = 0.83 arcsec) on 2004 December 11. From PSF-fitting photometry, we infer  $m_{\rm B} - m_{\rm A} = 1.22 \pm 0.01$  mag (1 $\sigma$ ), which translates into  $\Delta m_{\rm BA} = 1.22 \pm 0.0$  mag and A/B = 3.08 ± 0.08. The *r*-band flux of *B* is not corrected by a possible contamination by the lens galaxy light (the *r*-band correction will be significantly smaller than the 60-mmag offset in the *R* band, since the *r* Sloan filter does not transmit light at wavelengths of 700–900 nm), and the *r*-band magnitude difference is taken as the magnitude difference in

the *R* band (this is a reasonable assumption because the r - R colours of both components are similar). We note the agreement between the SDSS flux ratio at day 2250 (first filled triangle in Fig. 2.3), and the Maidanak/BroCam value of A/B around day 2300 (filled circle next to the first filled triangle in Fig. 2.3).



**Fig. 2.3.** Time evolution of the optical continuum (red wavelengths) flux ratio of FBQ 0951+2635. We show a previous measurement from NOT R-band frames (filled square at day 800), as well as new Maidanak/BroCam R-band estimates (filled circles) and two additional results from SDSS and Maidanak/SI 600 Series frames (filled triangles). We also show the lower limit of the Mg II flux ratio (base of the vertical arrow). The open circles and dashed lines are explained in the main text.

Maidanak observations in 2007 were done with the Spectral Instruments (SI) 600 Series CCD camera. It is a 4096 × 4096, 15-µm pixel CCD. The rectangular pixels have angular scales of 0.303 arcsec (along the horizontal axis) and 0.258 arcsec (along the vertical axis). The camera gain is  $1.45 \text{ e}^- \text{ADU}^{-1}$  and the readout noise is 4.7 e<sup>-</sup>. These Maidanak/SI 600 Series frames belong to a private archive that is managed by several teams. To test the time evolution of A/B in recent years, we use the three *R*-band frames that were taken on 2007 April 13. After applying a

60-mmag correction to the fluxes of *B*, the flux ratio at day 3104 is A/B =  $3.17 \pm 0.10$  (last filled triangle in Fig. 2.3).

Paraficz et al. (2006) suggested two possible *R*-band microlensing gradients over days 50–1000 (see middle and bottom panels in fig. 2 in [111]). However, the second solution (0.077  $\pm$  0.007 mmag d<sup>-1</sup>) seems more consistent with the analysis presented in [109], so we only consider this early microlensing slope. Authors of [109] found a flux ratio A/B =  $2.65 \pm 0.02$  ( $\Delta m_{BA} = 1.06 \pm 0.01$  mag) when they focused on the last part of the NOT brightness records, i.e. around day 800 (see the filled square in Fig. 2.3). This is fully consistent with the second linear fit in[111] (see bottom panel in fig. 2 of that work), which indicates  $\Delta m_{BA} = 1.06$  at day 800. Alternatively, the extrinsic variability could be due to two consecutive short-term microlensing events before day 700. While the first event was fitted to a Gaussian, it is also apparent the possible existence of a second event (see data point distribution between days 400 and 600). From day 700 to day 1000, the observations are consistent with constant microlensing because almost all error bars cross the zero level. If the two rapid events with similar amplitude are true features, no gradient is required to account for the observations. The two alternatives and their extrapolations to more recent epochs are depicted in Fig. 2.3. The inclined dashed lines represent the gradient of  $0.077 \pm 0.007$  mmag d<sup>-1</sup> (absence of short-term events). The horizontal dashed lines are the lower and upper limits associated with the second scenario: absence of a gradient, but presence of short-time-scale fluctuations with amplitude of about 60 mmag (flaring behaviour of *B*).

As we can see in Fig. 2.3, the Maidanak measurements from day 2600 to day 3100, i.e. the last two filled circles and the last filled triangle, are consistent with the first microlensing scenario. However, the most recent behaviour of A/B does not imply the presence of a long-term gradient lasting ~10 years. The whole set of results within the ~3000-d monitoring period suggests the existence of a long-time-scale microlensing fluctuation, which had a bump between day 1200 and day 2200,

and has remained almost flat during the last years. The bump consists of a significant increase (in A/B) of about 15 per cent followed by a shallow decrease. As the flux ratio is measured accurately, the signal-to-noise ratio for the prominent increase is  $\sim$ 3–5. When a source crosses a microlensing magnification pattern, the associated light curve may have a complex structure including different gradients at different epochs (e.g. Wambsganss 1990). We also point out that our new flux ratio estimates clearly disagree with the second scenario (see horizontal dashed lines in Fig. 2.3), which is based on the exclusive production of short-term ( $\sim$  months) fluctuations.

#### §2.3. Follow-up in the i SLOAN passband

We included FBQ 0951+2635 as a key target in our Liverpool Quasar Lens Monitoring (LQLM) programme [15,116]. The optical frames were taken with the LRT3 between 2007 February 6 and 2007 May 31. Each observation night consisted of five exposures of 100 s in the *i* band, using a dither cross pattern (see observation summary in Table 2.1). The LRT pre-processing pipeline included bias subtraction, trimming of the overscan regions and flat fielding. These preprocessed frames are publicly available on the Lens Image Archive of the German Astrophysical Virtual Observatory. We initially remove all frames that either are characterized by an anomalous image formation or have large seeing discs (FWHM > 2 arcsec in the frame headers). Later, we carry out cosmic rays cleaning and defringing. In the last step of the whole pre-processing procedure, all frames in each night ( $\leq$ 5) are combined, i.e. they are aligned and then averaged.

We create a stacked frame (consisting of the combination of the best exposures) to better detect the lens galaxy and to subtract its light in the combined frames. This represents a total exposure time of 4.4 h. The stacked image is characterized by FWHM = 1.17 arcsec and a very high signal-to-noise ratio (S/R = 413), where FWHM and S/R (aperture radius of  $2 \times$  FWHM) are measured on the S1 and S3 stars, respectively. In the i band, S3 has a magnitude similar to that of

the faintest quasar component *B*. Unfortunately, neither our best-combined frames in terms of FWHM and S/R nor our deep stacked frame leads to detection of the lens galaxy. Moreover, we obtain meaningless results by using constraints on the relative position and brightness profile of the galaxy (taken from [109]). The 2D signal from the very faint galaxy seems to be strongly affected by the photon noise of the quasar components and background, so we cannot resolve that signal in our frames. Thus, our instrumental photometry pipeline [115] only fits the instrumental fluxes of both components, which are described by stellar-like objects (PSF-fitting photometry, using S1 as PSF star). Once the instrumental photometry is done, we obtain calibrated and corrected brightness records of A, B and field stars. The transformation (calibration–correction) pipeline incorporates zero-point, colour and linear inhomogeneity terms, and the final magnitudes are given in the SDSS photometric system [115]. This transformation software is exclusively applied to the combined frames with FWHM < 1.5 arcsec and S/R > 50, i.e. on 22 out of 52 observation nights.

The top panel in Fig. 5 displays the LQLM light curves of the lensed quasar (A and B components) and the S3 star. Filled circles, filled triangles and open circles trace the behaviours of A, B (shifted by -0.9 mag) and S3 (shifted by -0.6 mag), respectively. There must be some diffuse contamination by the lens galaxy light, so the B fluxes in Fig. 2.4 (see Table 3 of [1]) are greater than the true ones. From the standard deviation of mS3 over the whole monitoring period, we infer a typical error in the S3 fluxes of 17 mmag (see open circles and associated error bars). Hence, the typical uncertainty in the B fluxes should be larger than 17 mmag, since B is as bright as S3 (~18.5–18.6 mag), but it is located within a crowded region. Unfortunately, even using frames with (FWHM) ~ 1.2 arcsec and  $(S/R) \sim 80$ , we cannot achieve 1–2 per cent level photometry for B (some deviations between adjacent nights are relatively large).



**Fig. 2.4.** LQLM i-band light curves of FBQ 0951+2635. Left: A (red circles), B - 0.9 mag (green triangles), and S3 - 0.6 mag (open circles). Right: comparison between the A master record (circles) and the time- and magnitude-shifted light curve of B (triangles).

Four pairs of adjacent fluxes of A show significant scatters around the mean values (in the top panel in Fig. 2.4, we draw four lines joining the members of each pair). For this reason, they are grouped (by computing four mean fluxes at the corresponding mean epochs) to obtain a master light curve, i.e. an accurate (smooth) trend that reliably describes the underlaying short-time-scale variability. This master curve is depicted in the bottom panel of Fig. 2.4 (filled circles). Before the gap, the quasar has a small level of activity. However, after the gap, there is a 60-mmag gradient lasting about 30 d. Thus, the *R*- and *i*-band records exhibit similar variability levels on a timescale of  $\sim 2$  weeks. To gain perspective on the

nature of the observed variability in the i band (A component), the B light curve is shifted in time (using the 16-d time delay by [109]), and then is compared to the master light curve of A.

The new LQLM light curves of FBQ 0951+2635 allow us to measure an accurate flux ratio (corrected by the time delay) in the *i* band:  $A/B_* = 2.74 \pm 0.02 (1 \sigma)$ . We do not measure the *i*-band flux ratio A/B around day 3100 but the contaminated ratio  $A/B_*$ , where  $B_* = B + G$  and G is a contribution due to the lensing galaxy.

We derive an average magnitude difference of 1.094 mag between the timeshifted record of B and the master curve of A (using bins with semi-size of 2–4 d). The time- and magnitude-shifted light curve of B is also included in the bottom panel of Fig. 2.4 (filled triangles). The typical error in the S3 fluxes is used as a lower limit for the uncertainty in the photometric measurements of B (see above). We find a clear agreement between the master curve of A and the adjacent fluxes of B, when these last fluxes are properly shifted in time and magnitude. Thus, there is no evidence of short-time-scale extrinsic variability (due to microlensing or another phenomenon) in our *i*-band observations.

The high-quality SDSS2 *i*-band frame of FBQ 0951+2635 at day 2250 (FWHM = 0.75 arcsec and S/R = 99) leads to a flux ratio  $A/B_* = 2.8 \pm 0.1 (1 \sigma)$ , in good agreement with the LQLM estimation. We note that  $B_* = B + G$  (see above), and the uncertainty in the SDSS flux ratio incorporates both photometric and simultaneity errors. This last error is associated with the use of fluxes at the same time of observation, i.e. without time delay correction (see above). There is no evidence of an appreciable evolution in  $A/B_*$  over days 2250–3100. This i-band result agrees with the Maidanak-SDSS recent trend of A/B in the *R* band. Comparing the flux ratios in both optical filters, we may derive that the lens galaxy light in the i band produces a plausible contamination of the B component in such filter (~130 mmag).

## §2.4. Observations and light curves of lensed images in the double quasar SBS1520+530

The broad absorption line (BAL) quasar SBS 1520+530 ( $z_q = 1.855$ ) was discovered in 1997 as a gravitationally lensed double quasar with an angular separation of 1". 56 [117]. The lensing galaxy was detected using infrared adaptive optics imaging at the Canada-France-Hawaii Telescope [118]. Faure et al. in [119] observed the lensing galaxy with the Hubble Space Telescope. [66] finally succeeded in obtaining both the redshift  $z_{gal} = 0.717$  (consistent with absorption lines first found in [117]) of the lensing galaxy with a Keck obervatory spectrum and the time delay of  $\Delta t = (130\pm3)$  days between the two quasar images using monitoring data from the Nordic Optical Telescope (NOT). For this, [66] obtained an almost gapless light curve of the object of about 800 days between February 1999 and May 2001. An almost continuous coverage of the light curve of SBS 1520+530 is made possible by its high declination. Further photometry obtained at Maidanak observatory on the system was published in [120].

Interestingly, [120] found that by simply shifting the light curve of image B backward by 130 days and correcting for the magnitude difference  $\Delta m = 0.69$  mag of the images made the quasar light curves align only approximately. They obtained a better match by allowing for an additional linear trend, and by correcting for faster variations using an iterative scheme [123]. Authors of [66] interpret these additional variations to be probably due to microlensing variability.

SBS 1520+530 thus is one of the prime targets for microlensing studies since it provides at once the prospects for long, uninterrupted light curves with a known, relatively short time delay and known microlensing variations. For this reason we decided to continue the optical monitoring of this system.

We report here on our observations of SBS 1520+530 (Fig. 2.5) with the 1.5m AZT-22 telescope in Maidanak between 2003-2008. In these intervals we observed the object almost daily whenever the weather permitted it. The V and R-filters provide photometry in the Johnson-Cousins system. We observed on average

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four frames per night with an exposure of 3.5 min in V and 3 min in R with a median seeing of 1.1 arcsec.



2.5. Fig. R-band image of SBS1520+530 obtained on May 7, the 2003. The quasar images, reference stars S3 and S4, and the star we use for the PSF are labelled. The field size is 2.8 arcmin  $\times$  3.5 arcmin. The area marked with the box is shown in Fig. 2.6.



**Fig. 2.6.** R-band zoom of the central part of SBS 1520+530. The field size is 30 arcsec × 25 arcsec. North is up and East is to the left. The bright foreground star next to the two quasar images A and B is saturated in the centre. S1 and S2 are also foreground stars. The lens galaxy ( $m_{R,gal} \approx 21.6$  mag, [118]) is too faint to be seen in this image.

A special property of the SBS 1520+530 system is its location within 14 arcsec of a bright 12<sup>m</sup> star (Fig. 2.6). This is great for adaptive optics studies of the system because a bright reference star is at hand [118]. For photometry, however,

the bright halo and the diffraction pattern caused by the star on the CCD needs to be subtracted carefully.

In order to do this, we followed the method described in [120]. We extracted the western half of the star and subtracted it from the eastern part where SBS 1520+530 is situated. This procedure also efficiently subtracts the light due to the horizontal diffraction spike that extends towards the double quasar.

Photometry on the quasar components A and B was performed using the DAOPHOT package [87]. We chose this method because it is well-suited to the mildly asymmetric and time-variable AZT-22 point spread function (PSF). Because of the variable PSF it was not possible to use the image subtraction technique [123,124]. The quoted error bars are the standard  $1\sigma$  errors determined by DAOPHOT.

Since there are four point sources (the quasar components A and B, and the stars S1 and S2) crowded in a rather small region, we had to fit the positions and magnitudes of all four components at the same time. Absolute quasar magnitudes were calibrated using the brightness of the reference star S3 (see Fig. 2.5) in the *V*-band ( $m_V = 17.37 \pm 0.02$  mag) and *R*-band ( $m_R = 17.18 \pm 0.02$  mag) determined in [120]. The star we used as a template to model the point spread function in DAOPHOT is marked with "PSF" in Fig. 2.5.

In our analysis we ignore the presence of the lensing galaxy near image B because it is too faint to be detected in our images ( $m_{R,gal} = 21.6 \text{ mag}$ , [118]). If all of the galaxy light contributes to our magnitude estimate of quasar image B, the measured brightness would increase by a constant offset of 0.08 mag (assuming  $m_{R,B} = 18.8 \text{ mag}$ ).

In Fig. 2.7. we present the results of the R-band photometry of SBS 1520+530 obtained in 2003-3004. The light curves of the two quasar components A and B are plotted together with the light curve of an additional reference star S4 (see Fig. 2.5,  $m_R = 15.76$  mag). This shows that both S3 (which is used for the absolute magnitude calibration) and S4 do not vary.



**Fig. 2.7.** R-band light curves of the A and B lensed images in SBS 1520+530 and the reference star S4. For clarity the B magnitude was shifted up by -0.4 mag and S4 was shifted by +2.4 mag. The dotted vertical line indicates the day with the Julian Date-2 452 000 = 880 when the mirror was cleaned [46].



**Fig. 2.8.** V–R colour curve for the quasar images A and B. The best-fitting value for each quasar image is plotted with a dashed line [46].

Both quasar components show low-amplitude  $\Delta m \approx 0.1$  mag variations on time scales of about 100 days. As we will show in the next section, the overall similarity of the two quasar image light curves is due to a gradual brightness

decrease of the lensed quasar. On the day with Julian Date-2 452000 = 880 the telescope mirror was cleaned. This led to a large sensitivity improvement that visibly improved the accuracy of the quasar brightness measurements, especially for the fainter image B.

In Fig. 2.8 we show the difference between our V-band and R-band light curves of SBS 1520+530. We find an average V - R colour  $m_V - m_R = 0.15$  mag for image A and  $m_V - m_R = 0.18$  mag for image B. We do not find any evidence for significant colour variations during our observing interval. The small difference  $\Delta(V - R) \approx 0.03$  mag of the V – R colour between the quasar images indicates the presence of a small level of differential reddening along the light paths.



**Fig. 2.9.** Total R-band light curves of the two quasar images A and B in SBS 1520+530 obtained in 2003-2008 in Maidanak observatory.

At the first stage of research (2003-2004) we calculated time delay in this system and revealed presence of microlensing. Later our subsequent studies have confirmed our conclusions. In Fig. 2.9 we present total light curve of A and B lensed images of SBS1520+530.

#### §2.5. Microlensing in the double quasar SBS1520+530

Before to start the microlensing analysis in SBS1520+530, let us briefly discuss the problem of time delay. Authors of the paper [66] based on the data of observations in 1999-2001 were able to find time delay  $130\pm3$  days and the magnitude difference between the components was equal to -0.69 mag. Later, we confirmed this time delay in the paper [46] based on observations in 2003-2004, but the magnitude difference between the components (A – B) increased significantly and became equal to -0.83 mag. In the remainder of this paper we will use this time delay for our data.



**Fig. 2.10.** R-band light curves of the two quasar images A (filled circles) and B (open circles). The B light curve has been shifted by -130 days, as indicated by the arrow, and by -0.83 mag [46].

In the top panel of Fig. 2.10 we show the quasar A and B light curves in one plot, where image B was shifted to the left by the time delay of 130 days and up by the magnitude offset of -0.83 mag. The composite light curve has no large gaps. It shows that the quasar has been going through a series of three small  $\Delta m \approx 0.1$  mag brightness variations that each lasted about 100 days.

In Fig. 2.11 we show the same curves as in previous plot for all period of our observations. Also here we plotted magnitudes by Burud et al. [66] taking into

account our time delay and (A-B) magnitude difference. Here we see a continuous changing not only in visible brightness of the components, but also in the difference between lensed components. This indicates a long process of microlensing.



**Fig. 2.11.** The same curves as in Fig.2.12 for all period observations of SBS1520+530 together results from [66].

Microlensing in the lens galaxy would only affect one of the light paths to the quasar and could thus be detected as a residual light curve difference (e.g., [121,122]). In order to study whether microlensing variations are present in our data, we calculated the difference between the two observed light curves for the time delay  $\Delta t = 130$  days found by Burud et al. [66] and the magnitude offset of  $\Delta m = -0.83$  mag.

To calculate the difference light curve, quasar B was shifted in time and magnitude. The rest of the procedure is identical to the one described earlier. We calculated the difference by linearly interpolating the light curves of quasar A or B. The light curves were interpolated whenever the gap was less than 40 days. No difference was calculated for larger gaps. The error bars were added in quadrature.

In their earlier data taken between February 1999 and May 2001, authors of [66] did find a difference between the light curves of the two quasar images in this system. Applying our procedure to the light curves in their Table 2, we can also calculate the difference light curve of their data. The result is shown with in Fig. 2.12 together with our difference light curve.



**Fig. 2.12.** Composite of the difference light curves based on our Maydanak data and the data published by Burud et al. [66]. We can see the linear trend in (A-B) magnitude difference.

The Fig. 2.12 shows that [66] observed a coherent and highly significant difference light curve with a maximum amplitude of  $\Delta m \approx 0.08$  mag. Since the time of the observations in [66] the magnitude difference between the quasar images has increased by  $0.14 \pm 0.03$  mag. [66] already identified a linear trend in their data. Our data are consistent with this linear trend having continued until the epoch of our observations.

Although the exposure times for the Maidanak and data in [66] are similar, the error bars of the Maidanak data are larger than the error bars obtained in [66]. The main reason for this are the different telescope apertures (1.5 m at the AZT-22
vs. 2.5 m at the NOT). In addition, however, the AZT-22 transmission was reduced by about one magnitude before the mirror was cleaned on Julian Date-2452 000 = 880 (see Fig. 2.8).

We note that the long-term variation of the difference light curve may even be slightly larger than shown in this plot because some light from the lensing galaxy could be included in our magnitude estimate of image B (see Sect. 3). If all of the lensing galaxy light were included, the Maidanak difference light curve would have to be shifted upward by a constant offset of -0.08 mag to correct for the galaxy contribution.

## §2.6. Summary and discussion on Chapter II

A previous analysis showed the existence of extrinsic variability of the observer-frame optical continuum in the gravitationally lensed double quasar FBQ 0951+2635 [111]. The two quasar components (A and B) cross two different regions of the lensing galaxy, so distributions of collapsed objects could affect one (or both) of the light curves and then produce extrinsic variations. This microlensing hypothesis is more plausible for the B component because it crosses the central region of the galaxy (see [109]). [111] reported on two possible microlensing variabilities at red wavelengths in the 1999–2001 period: short-time-scale events (B flares having a duration of months) or a gradient lasting a few years. Both alternatives can account for the 1999–2001 *R*-band observations, and this work sheds light on the dominant kind of microlensing fluctuations occurring in FBQ 0951+2635.

We analysed new Maidanak R-band images taken during the 2001–2006 period (a 6-yr homogeneous monitoring), and a few complementary frames in the rR bands taken in 2004 and 2007 (SDSS and Maidanak/SI 600 Series archives). The Maidanak-SDSS flux ratios (A/B) in the red part of the optical continuum are inconsistent with the absence of long-time-scale (~years) gradients and the continuous production of B flares lasting a few months (short-time-scale

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microlensing events). If this last scenario were true, all data points in Fig. 2.3 would be distributed between the two horizontal dashed lines. The whole set of flux ratios (from 1999 to 2007) favour the existence of a long-time-scale microlensing fluctuation, so long-time-scale gradients seem to be the dominant microlensing variations. While A/B shows a bump during the 2003–2004 period, it is almost constant from late 2004 to the middle of 2007. This last quasi-stationary behaviour is supported by additional data in the *i* band (from new LQLM images taken in 2007 and the SDSS archive frame in that filter). [46] and [111] also found a long-time-scale microlensing variation in the doubly imaged quasar SBS 1520+530. Although the existence of rapid flares in the early years of monitoring (1999–2001) cannot be ruled out, these hypothetical B flares are not produced in a continuous way. Short-time-scale microlensing is not detected in the Maidanak variability study over the first semester of 2004. The LQLM data in the *i* band also indicate the absence of short-time-scale events in 2007. Apart from the 1999–2007 optical data, FBQ 0951+2635 has recently been observed in X-rays [127]. However, the X-ray flux ratio has a large uncertainty, and it is not useful for comparison with our estimates in the optical continuum.

If microlensing in the B component is the physical origin of the long-timescale fluctuation, this fluctuation represents a progressive demagnification of B followed by a quasi-stationary evolution in more recent epochs. The optical continuum flux ratio at red wave-lengths is always below the Mg II flux ratio ([109], see also Fig. 4), which is usually assumed to be weakly affected by microlensing. Thus, the recent microlensing magnification of B would be relatively weak, but still appreciable. The microlensing peak (maximum magnification of B) would have taken place before the discovery of the lens system in [105]. Strictly speaking, the continuum flux ratio refers to the flux ratio in the R band, i.e. a spectral region containing both the red continuum and the Mg II emission line. However, this line only contributes a few per cent to the broad-band fluxes (see fig. 4 in [105]). It can be shown that the line-corrected ratio is very similar to the ratio from total fluxes in the R band. We also note that A/B is not corrected by any extinction factor, so this flux ratio might be a biased estimator of the lens (macrolens + microlens) magnification ratio. For example, [128] suggested the relative extinction of A by dust in the lensing galaxy. This dusty scenario leads to a true lens magnification ratio greater than A/B.

We point out that new flux ratios at a large collection of wave-lengths should be key tools to discuss dust extinction and microlensing in the lensing galaxy (e.g. [40,128,129,130,131,132]). Moreover, FBQ 0951+2635 must be imaged in the rR bands during the next years (several times per year, with a two-week separation between consecutive observations). This modest monitoring programme will lead to draw the future evolution of A/B, as well as to obtain relevant information on the structure of both the source quasar and the intervening galaxy (e.g. [112,133,134]).

As for the SBS 1520+530, we have presented V-band and R-band photometry of the CCD-images taken at Maidanak Observatory in the years 2003 - and 2008. Our light curves show the quasar continuously reduces its brightness, but at times undergoes a series of brightness variations with  $\Delta m \approx (0.1-0.2)$  mag, each of which lasts about 100-500 days.

The V – R colour of the quasar is consistent with being constant during the observed period. Using linear interpolation of the quasar light curves, we determined earlier in the paper [46] time delay values and confirmed  $\Delta t = (130 \pm 3)$  days found in [66]. Image A is leading, which is also consistent with lens models of the system [66,119,120,135].

Any variable difference between the light curves of SBS 1520+530 can be interpreted as gravitational microlensing because other changes of the source would be visible in both quasar images, delayed by the time delay. In addition to the microlensing variability on short time-scales found in [66], our data show that there are also variations on longer time-scales of more than 10 years. This overall

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level of microlensing variations in SBS 1520+530 appears comparable to variations seen in other lens systems (e.g., [67,136,137]).

An exciting prediction for the microlensing effect of quasars is the colourdependence of the microlensing light curve in the vicinity of caustics [40]. In such a situation the difference between the V and the R light curve could provide valuable clues to the source structure of the quasar. We will continue to observe SBS 1520+530 from Maidanak observatory because frequent sampling of the source remains crucial to derive limits on microlensing variability. If colour variations associated with microlensing could be proven in this system, there would be a strong case for parallel spectral observations of the quasar.

Since microlensing currently remains the only technique with the promise to scan the continuum emission regions of quasars on micro-arcsecond scales, SBS1520+530 should be viewed as a prime target because of the combination of known quasar variability and the at least occasional occurrence of microlensing diagnostics at the same time.

# CHAPTER III. QADRUPOLE LENSED QUASARS PG 1115+080 AND B1422+231: VARIATIONS OF THE LIGHT CURVES AND TIME DELAYS VALUES

## §3.1. Observations of the PG1115+080

The quadruply imaged quasar PG 1115+080 is one of the most promising candidates both to investigate the dark matter problem and to determine the  $H_0$ value from measurements of the time delays between the image components. The source with a redshift of  $z_q = 1.722$  is lensed by a galaxy with  $z_g = 0.31$ [138,139,140], which forms four quasar images, with an image pair A1 and A2 bracketing the critical curve very close to each other. It is the second gravitationally lensed quasar discovered over a quarter of century ago, at first as a triple quasar [141]. Authors of [142] were the first to resolve the brightest image component into two images separated by 0.48 arcsec. Further observations have provided positions of quasar images and information about the lensing object [139,143,144,145,146], which allowed construction of a macrolens model (e.g. [147]). In particular, the authors of [148] have shown that the observed quasar image positions and fluxes and the galaxy position can be fit well by an ellipsoidal galaxy with an external shear rather than by only an ellipsoidal galaxy, or by a circular galaxy with an external shear. They noted that a group of nearby galaxies detected in [143] could provide the needed external shear.

Observations of PG 1115+080 were started at the 1.5-m telescope of the high-altitude Maidanak Observatory in 2001. An image scale of 0.26 arcsec/pixel was available at the f/8 focal plane with a scientific BroCam CCD camera having a SITe ST 005A 2030  $\times$  800 chip. The CCD images were usually taken in series consisting of two to ten frames for the R filter and of two to six frames for V and I. To provide higher photometric accuracy, we averaged the values of magnitudes estimated from individual frames. The seeing varied from 0.75 to 1.3 arcsec of the images of the reference stars B and C according to designation in [144]. The

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analysis of photometry shows no significant dependence of the photometry errors on seeing, excepting the FWHM noticeably exceeding 1.3 arcsec. Occasional frames with such values of the FWHM were excluded from processing.



**Fig. 3.1.** PG 1115+080 from observations in filter R with the 1.5-m telescope of the Maidanak Observatory. The image was obtained by averaging of six frames from a series obtained in 2004 February 24, with a subsequent Richardson–Lucy processing.

In Fig. 3.1 we show one of the best images of PG 1115+080 obtained through the R filter. For better view, the image was restored with an algorithm similar to that proposed in [149] in optics and independently in [150] (the Richardson–Lucy iterative method).

The algorithm of photometric image processing is similar to that applied to the photometry of Q2237+0305 and described in great details in [151]. The light curves of PG 1115+080A1, A2, B and C in filter R for the time period from 2001 April to 2006 June are shown in Fig. 3.2 here and Tables 3,7 and 8 in our paper [4].

Unfortunately, observations were not carried out in 2003, and the data are very scanty for the 2001 and 2002 seasons in all the three filters. The most numerous data were obtained in filter R, especially in 2004 (23 nights), 2005 (27 nights) and in 2006 (24 nights). The data demonstrate noticeable variations of the quasar brightness, with the total amplitude reaching approximately 0.4 mag in 2004 - 2006, and smaller amplitudes of about 0.05 mag on a time-scale of 2 months, which are clearly seen in all the four light curves in 2004. The time delays

between the light curves of the C and B, C and A1 (or A2) images can be easily seen from a simple visual inspection of the R light curves, therefore, the data obtained in 2004 - 2006 in filter R seem to have good prospects for obtaining reliable estimates of the time delays in PG 1115+080.



**Fig. 3.2.** The light curves of PG 1115+080 A1, A2, B, C from observations in filter R with the 1.5-m telescope AZT-22 in 2001, 2002, 2004, 2005 and 2006.

Thus, our photometry in filter R has demonstrated the applicability of the light curves obtained to determine the time delays. As compared to the data used in [59] and [62], we were lucky to detect the quasar brightness variations with an amplitude of almost a factor of 3 larger and with rather well-sampled data points within every season of observations. In addition, the accuracy of our photometry has made it possible to confidently detect flux variations with an amplitude as small as 0.06 mag, that can be seen in the data of 2004.

Also observations of PG 1115+080 in filter R during the same time periods in 2004–2006 should be mentioned here [164]. We have made use of their photometry presented in their table 3 to compare with our light curves. Variations of the quasar brightness which allowed us to determine the time delays are seen in their A1+A2 light curve quite well, but become undetectable in the B and C light curves because of a much larger scatter of the data points. Thanks to the spatially resolved photometry of the A1 and A2 image pair in filters V, R and I, our data have made it possible to measure flux ratios for these components for five seasons of observations, and to study their behaviour in time and in wavelength. As is noted above, deviations of flux ratios in quasar macro-images from the theoretical predictions (flux ratio anomalies) are presently believed to be diagnostic for detection of substructures in lensing galaxies, which may represent the dark matter.

## §3.2. Calculation of the time delays in PG1115+080

The ideology of methods to determine the time delays between two image components from their light curves is simple enough and obvious. A common feature of all known methods of time-delay measurements is the use, in one way or the other, of the cross-correlation maximum or mutual dispersion minimum criteria, while the methods may differ in the algorithms of the initial data interpolation. Analysis of the light curves of quasar images in pairs can be also applied when a lens consists of more than two images. But, however, another approach seems to be more promising in this case. The model source light curve can be determined from a joint analysis of light curves of all image components. The individual time delays of the components are then determined with respect to this model source light curve jointly from a corresponding system of equations. In some cases, this approach allows systematic variations in the light curves to be revealed and taken into account, such as those caused by, e.g., microlensing. A similar approach described earlier in [152] and [153] and used later in [59] was also applied to determine the time delays in Q2237+0305 [154].

The time delays in PG 1115+080 were determined for the first time in [62] to be  $23.7 \pm 3.4$  days between B and C, and  $9.4 \pm 3.4$  days between A1+A2 and C (image C is leading). The authors of [59] re-analysed their data using another algorithm and reported  $25^{+3.3}_{-3.8}$  days for the time delay between B and C, and this is quite consistent with  $23.7 \pm 3.4$  days from [62]. However, the other time delays,

and hence the time delay ratio  $r_{ABC} = \Delta t_{AC}/\Delta t_{BA}$  differ significantly:  $r_{ABC} = 1.13^{+0.18}_{-0.17}$  as calculated in [59] and  $0.7 \pm 0.3$  according to [62]. Since 1997, just these values, either the first or the second ones, were being used to constrain the PG 1115+080 model and determine the Hubble constant.

Determination of the time delays has generated a flow of models for the system [62,146,148,155,156,157,158,159,160,161,162,163], all illustrating how strongly the estimated value of H0 depends on the adopted mass profiles of the lens galaxy for the given values of time delays.

The detailed analysis of the uncertainties in determining Hubble constant from the time delay lenses can be found in e.g. [164,165,166], where the paths to eliminate or at least to lessen the uncertainties have also been outlined. Authors of [164] indicated, in particular, the importance of improving the accuracy of time delays for PG 1115+080.

The R light curves taken from the 2004–2006 data (Fig. 3.2) clearly demonstrate their applicability to determine the time delays. As compared to the data used in [59] and [62], we were lucky to detect the quasar brightness variation with an amplitude of almost a factor of 3 larger, and with rather well-sampled data points within every season of observations. In addition, the accuracy of our photometry has made it possible to confidently detect flux variations with an amplitude as small as 0.05 mag that can be seen in the data of 2004.

As was noted above, a necessity to properly interpolate the unevenly sampled data points in the light curves under consideration is one of the main technical problems in determining the time delays. A variety of interpolating functions and algorithms is used, such as polynomials of various power, Legendre polynomials etc. In some cases, the low-frequency splines provide good results. But unfortunately, all these interpolation procedures do not contain any physical meaning. Meanwhile, the Fourier spectra of quasar variability are known to be rapidly decreasing functions [167], and this is naturally explained by the finite physical sizes of quasars. In particular, the quasar size is known to play the role of a smoothing factor in microlensing light curves, as is clearly seen from simulations. Therefore, we tried to find such an algorithm to represent the source quasar light curve, which would take into account the expected frequency characteristics of the quasar variability and allow a relevant smoothing of the observational data. To do this, it was natural to address an expression that is well known in optics, radio engineering and theory of information as the sampling theorem. According to this theorem, a signal with a bounded spectrum can be represented accurately enough by a function:

$$f(t) = \sum_{k=-\infty}^{\infty} a(k\Delta t) \operatorname{sinc} \left[ \pi \frac{t - k\Delta t}{\Delta t} \right].$$
(3.1)

Here, the function  $\operatorname{sinc}(x)$  is specified as  $\operatorname{sinc}(x) = \operatorname{sin}(x)/x$ , and  $a(k\Delta t)$  are samples of f(t) taken at a mesh with a step  $\Delta t$ , which is determined by a boundary frequency  $\Omega_{\text{bnd}}$  of the function f(t) spectrum:  $\Delta t = 1/(2 \Omega_{\text{bnd}})$ . Since, as was noted above, Fourier spectra of quasar variability are rapidly decreasing functions, we may apply the sampling theorem to reproduce the quasar light curve. We used this approach earlier to analyse the statistics of microlensing brightness variations in Q2237+0305.

The same algorithm was used in this work to represent the model for the source light curve. Since the time delay between the light curves of images A1 and A2 is very short 168,169], their fluxes were summed to form a single curve, which we will call the A light curve. Thus, we may write the following functional for three light curves:

$$\Phi(\Delta t, \tau_0, \tau_1, \tau_2) = \frac{1}{3N} \sum_{j=0}^{2} \sum_{i=0}^{N} \frac{\left[m_j(t_i) + dm_j - f(t_i, \Delta t, \tau_j)\right]^2}{\sigma_j^2(t_i)}$$
(3.2)

where  $m_j(t_i)$  are the data points in the light curve of the  $j^{\text{th}}$  image at the time moments  $t_i$ ,  $dm_j$  and  $\tau_j$  are the shifts of the corresponding light curves in stellar magnitude and in time, respectively, *N* is the number of points in the light curves,  $\sigma_j^2(t_i)$  are the photometry errors and, finally,  $f(t_i, \Delta t, \tau_j)$  is an approximating function (3.1).

We adopted  $dm_0 = 0$  and  $\tau_0 = 0$  in our calculations, that is we fitted the light curves of the two other images to the A light curve and, thus,  $dm_1$  and  $dm_2$ are the magnitude differences A – B and A – C, respectively. At given values of  $\tau_1$ and  $\tau_2$ , we minimize  $f(\Delta t, \tau_1, \tau_2)$  in  $dm_j$  and in coefficients  $a(k\Delta t)$  of the sampling function. The values of the minimum of  $\Phi(\Delta t, \tau_1, \tau_2)$  were being looked for at a rectangular mesh  $\tau_1, \tau_2$  with a step of 0.5 d in preliminary calculations and of 0.2 d at a final stage. The values of  $\tau_1, \tau_2$  corresponding to the minimal value of  $\Phi(\Delta t, \tau_1, \tau_2)$ , were adopted as the estimates of the time delays  $\tau_{BA}$  and  $\tau_{AC}$ . The time delay  $\tau_{BC}$  is not an independent quantity in our method and can be determined as a linear combination  $\tau_{BC} = \tau_{BA} + \tau_{AC}$ .



Fig. 3.3. Distribution of  $\Phi(\Delta t, \tau_1, \tau_2) - \Phi_{\min}$  in the space of parameters  $\tau_{AC}$  and  $\tau_{AB}$ . The innermost contour corresponds to  $\Phi(\Delta t, \tau_1, \tau_2) - \Phi_{\min}$  equalling 0.0001, with every next level twice as much than preceding.

Fig. 3.3 shows a distribution of  $\Phi(\Delta t, \tau_1, \tau_2) - \Phi_{\min}$  in the space of parameters  $\Delta t_{BA}$  and  $\Delta t_{AC}$  calculated for parameter  $\Delta t = 0.12$  year. Thus, our estimates of the time delays that can be read out at the  $\Delta t_{AC}$  and  $\Delta t_{BC}$  axes against the centre of contours in Fig. 3.3 are  $\Delta t_{BA} = 4.4$  days and  $\Delta t_{AC} = 12.0$  days. The time delay  $\Delta t_{BC}$  is not an independent quantity in our method, and can be determined as a linear combination  $\Delta t_{BC} = \Delta t_{BA} + \Delta t_{AC}$ , i.e.  $\Delta t_{BC} = 16.4$  days.

Having calculated the time delays, we analysed the deviations  $\delta_j$  of the light curves of each image from the approximating function:

$$\delta_j(t_i) = m_j(t_i) + dm_j - f(t_i, \Delta t, \tau_j)$$
(3.3)

We revealed a linear trend in the deviations of the A light curve and small parabolic trends for images B and C. We interpreted these trends as the effects of microlensing, which are expected to be rather slow and weak. We then subtracted a half of these trends from the initial light curves of the components, obtained new estimates of the time delays and analysed the residual trends again. This procedure continued iteratively until the trends became insignificant. The final estimates of  $\tau_1$  and  $\tau_2$  are obtained with the linear trend of 0.0105 mag/yr subtracted from the image A light curve, and with the parabolic trend with the amplitude of  $\pm$  0.01 mag subtracted from the B and C light curves. Thus, our estimates of the time delays are  $\Delta t_{BA} = 4.4^{+3.2}_{-2.4}$  days,  $\Delta t_{AC} = 12.0^{+2.4}_{-2.0}$  days and  $\Delta t_{BC} = 16.4^{+3.4}_{-2.4}$  days, with the relationship  $\Delta t_{AC}/\Delta t_{BA}$ , more consistent with that determined in [59] than in [62].

To evaluate the errors of estimating the time delays and reliability of our estimates, we fulfilled a numerical simulation. We selected a function  $f(t_i, \Delta t, \tau_j)$  used to approximate our light curves with  $\Delta t = 0.08$  yr, as a model source light curve. The model light curves of the components were obtained by shifting  $f(t_i, \Delta t, \tau_j)$  by the proper time delays  $\tau_1$ ,  $\tau_2$  and magnitude differences, and by adding some random quantities to imitate the photometry errors. The estimates of these errors were obtained from the analysis of deviations of the observed data points from the approximating function resulting in the values of standard deviations 0.008, 0.016 and 0.011 mag for images A, B and C, respectively. Since the method we used might be susceptible to the mutual locations of data points in the actual light curves, we selected the model samples exactly at the same time moments as in the actual light curves. We simulated two cases:  $\Delta t_{BA} = 14.3$  days

and  $\Delta t_{AC} = 9.4$  days as determined in [62] and  $\Delta t_{BA} = 4.4$  days and  $\Delta t_{AC} = 12$  days (our result). We simulated 2000 random light curves synthesized as described above and calculated the resulting time delays using the procedure, which was exactly the same as in the analysis of the actual light curves. Admitting that we could be mistaken in the selection of the Nyquist interval, we fulfilled the model calculations with a more low-frequency function of the source brightness variations ( $\Delta t = 0.12$  yr). No systematic biases larger than 0.3 d in the estimates of simulated time delays  $\tau_1$  and  $\tau_2$  were revealed in both cases. The results of simulations were used to estimate the 95 per cent confidence intervals.



**Fig. 3.4.** The data points of the A, B and C light curves superposed with each other after shifted by the corresponding magnitude differences and time delays are  $\Delta t_{BA} = 4.4^{+3.2}_{-2.4}$  days,  $\Delta t_{AC} = 12.0^{+2.4}_{-2.0}$  days and  $\Delta t_{BC} = 16.4^{+3.4}_{-2.4}$  days obtained in this work. The parameter  $\Delta t$  of the approximating function (shown in a solid curve) is 0.12 yr.

It is interesting to note that using only the data of 2004, where a smallamplitude turn over in the light curves is detected, we obtained  $\Delta t_{BA} = 5.0$  days,  $\Delta t_{AC} = 9.4$  days and  $\Delta t_{BC} = 14.4$  days, consistent with the estimates obtained from the whole data set. But, however, simulation of errors for only this time interval demonstrates noticeably larger uncertainties, as compared to those calculated from the whole light curve. In the Fig. 3.4, the light curves of images A, B and C shifted by the corresponding time delays and reduced to image A in magnitude are shown for the approximating function parameter  $\Delta t = 0.12$  yr. As is seen, the data points for all the three images are very well consistent with each other and with the approximating curve (a behaviour of the approximating function within the gaps between three seasons of observations should be ignored).

To test our method for robustness and absence of systematics, and to estimate the accuracy inherent in our time delay measurements, we fulfilled a numerical simulation as described in detail in [2]. The simulated light curves of the components were obtained by shifting the approximating curve  $f(t_i, \Delta t, \tau_j)$  by the proper time delays  $\tau_1$ ,  $\tau_2$  and magnitude differences, and by adding random quantities to imitate the photometry errors. We simulated 2000 light curves synthesized as described above, and calculated the resulting time delays using the procedure, which was exactly the same as in the analysis of the actual light curves. The results of simulations were used to build the distribution functions for errors and to estimate the 95 per cent confidence intervals.

Table 3.1. The time delays (in days) for PG 1115+080.

Referenes	$\Delta t_{ m BA}$	$\Delta t_{ m AC}$	$\Delta t_{ m BC}$
[62]	$14.3 \pm 3.4$	9.4 <u>+</u> 3.4	23.7 ± 3.4
[59]	$11.7^{+2.9}_{-3.2}$	$13.3^{+2.0}_{-2.2}$	$25.0^{+3.3}_{-3.8}$
[2]	$4.4^{+3.2}_{-2.5}$	$12.0^{+2.5}_{-2.0}$	$16.4^{+3.5}_{-2.5}$

The final values of the time delays and the corresponding uncertainties are presented in Table 3.1, where they can be compared with the estimates reported in [59] and [62].

It is interesting to note that using only the data of 2004, where a smallamplitude turnover in the light curves is detected, we obtained  $\Delta t_{BA} = 5.0$  days,  $\Delta t_{AC} = 9.4$  days and  $\Delta t_{BC} = 14.4$  days, consistent with the estimates obtained from the whole data set. However, simulation of errors for only the data of 2004 demonstrates noticeably larger uncertainties, as compared to those calculated from the entire light curve.

The light curves of images A, B and C shifted by the corresponding time delays and reduced to image A in magnitude are shown in Fig. 2 for the approximating function parameter  $\Delta t = 0.12$  years. As is seen from this picture, the data points for all the three images are very well consistent with each other and with the approximating curve.

Thus we obtained the time delay values, which differ noticeably from those reported by [59] and [62] and used in a variety of models of many authors to derive the Hubble constant value. The largest differences are for  $\Delta t_{BC}$  and  $\Delta t_{BA}$ : our estimate of  $\Delta t_{BC}$  is a factor of 1.5 smaller, while for  $\Delta t_{BA}$ , it is almost three times smaller as compared to the results of [59] and [62]. Meanwhile, our values of  $\Delta t_{AC}$ are rather similar to those of Schechter and, especially, of Barkana.

**Table 3.2.** Time delay ratios  $\Delta t_{AC}/\Delta t_{BA}$  and  $\Delta t_{AC}/\Delta t_{BC}$  for PG 1115+080 as predicted by several lens models (the upper part of the table) and determined from the existing measurements of the time delays for the system (the last three lines).

References	$\Delta t_{ m AC}/\Delta t_{ m BA}$	$\Delta t_{\rm AC} / \Delta t_{\rm BC}$	
[62]	1.33-1.80	0.57–0.64	
[148]	1.35–1.47	_	
[155]	1.3	_	
[169]	1.3	0.56	
[170]	1.54	0.61	
[62]	0.66	0.40	
[59]	1.14	0.53	
[2]	2.73	0.73	

So we obtained the time-delay values, differing noticeably from those in [59] and [62], which are used in a variety of models of many authors to derive the Hubble constant value. Calculation of a new lens model or recalculation of some most popular ones to derive the new estimate of the Hubble constant would be well

beyond the scope of the present short communication. We would just note that the new values of time delays reported in this work must result in higher values of  $H_0$  as compared to those obtained with the previous time-delay values for PG 1115+080, thus decreasing a well-known diversity between the time-delay method of determining the Hubble constant and other methods.

#### §3.3. The A2/A1 flux ratios

The idea to detect substructures in lensing galaxies using the anomalies of flux ratios is based on fundamental relationships between coordinates and magnifications of the quasar images, which result from the general lens equation [171]. These relationships have been obtained for the first time in [172] and [173] for several 'smooth' distributions of lensing potential. In principle, the lens equation is capable of providing six independent relationships between the coordinates and magnifications for a quadruple lens, but only one of them can be checked with the data of observations. This is the well-known magnification sum rule for a source within a macrocaustic cusp, when three close images emerge: magnification of the central image must be equal to the sum of magnifications of two outer images [172]. When the source lies near a caustic fold, two images of the same brightness must arise. Since the absolute values of magnifications in macro-images are unknown (the unlensed quasar cannot be observed), authors of [174] proposed to use the dimensionless quantities

$$R_{\rm cusp} = \frac{|\mu_1| - |\mu_2| + |\mu_3|}{|\mu_1| + |\mu_2| + |\mu_3|} = \frac{F_1 - F_2 + F_3}{F_1 + F_2 + F_3}$$
(3.4)

for three images emerging when the source is in a caustic cusp, and

$$R_{\rm fold} = \frac{|\mu_{\rm m}| - |\mu_{\rm s}|}{|\mu_{\rm m}| + |\mu_{\rm s}|} = \frac{F_{\rm m} - F_{\rm s}}{F_{\rm m} + F_{\rm s}}$$
(3.5)

for the case when the source is at the caustic fold. Indices m and s in the second expression denote the images at the minimum and saddle points of the Fermat surface.

Ideally, cusp relation  $R_{cusp} = 0$  and fold relation  $R_{fold} = 0$  hold only when the source lies exactly at the caustic cusp or fold, respectively. In real lenses these relations hold only approximately. Authors of [175] and [176] fulfilled a detailed study of asymptotic behaviours of the cusp and fold relations and calculated probability distributions of  $R_{fold}$  values for several smooth lens models. The value of deviation of  $R_{cusp}$  and  $R_{fold}$  from zero can be regarded as a measure of probability for the lensing potential to have sub-structures on scales smaller than the separation between the closest images [175,176]. They warn, however, that for fold lenses, the observed violation of the fold relation may just mean that the source is far enough from a caustic fold.

It should be noted that, in principle, the observed anomalies of brightness ratios in images of gravitationally lensed quasars can be explained by other factors, such as microlensing by compact bodies and the effects of propagation phenomena in the interstellar medium (extinction and scattering, scintillations). These factors are studied in details in [177]. Those authors concluded that substructures of cold dark matter are the best explanation for the flux ratio anomalies in some quadruply lensed quasars. They reminded also that, as was stated for the first time in [174], the fluxes of highly magnified saddle images are very sensitive to small gravitational perturbations as compared to low-magnification images and, even more importantly, these perturbations bias the fluxes towards demagnification, as was also noted in [178].

Now we consider the behaviour of the flux ratios in time. The A1+A2 image pair in PG 1115+080 consists of a highly magnified minimum point image (A1) and saddle point image (A2) situated symmetrically with respect to the fold caustic very close to each other. According to theoretical expectations (e.g. [171]), the ratio of their fluxes must be close to 1. There are numerous measurements of the A2/A1 brightness ratio in PG 1115+080 made at different spectral ranges and at different epochs since 1980, which we tried to assemble in Table 3.3. Some of these data have been used in [179] to analyse a long-term history of the A2/A1 variations in the optical band and to compare with the X-ray data (see e.g. [162,179,180]). Fig. 2 in the paper [179] demonstrates changes in the A2/A1 optical flux ratio during the time period from 1980 to 2008, and much more dramatic changes of this ratio in X-rays. They noted that, according to all the observations since the system discovery, the A2/A1 flux ratio varied within 0.65–0.85. They did not specify the optical spectral bands for the data in their fig. 2, however, but it looks like they are for filter V.

period 1980–2000 from	all available us	ala.		
Date	A2/A2	Spectral	Instrument	Reference
	flux ratio	range		
1980 June	0.83	V	MMT	[142]
1981 April 30	1.00	В	CFHT	[144]
1983 March 8	1.00	V	CFHT	[144]
1984 March 26	0.95	В	CFHT	[144]
1985 March 16	0.75	В	CFHT	[144]
1985 March 19	0.79	V	CFHT	[144]
1986 February 19	0.79	V	CFHT	[139]
1986 February 19	0.80	R	CFHT	[139]
1986 February 19	0.79	В	CFHT	[139]
1989 April	0.68	Ι	CFHT	[181]
1991 March 3	0.66	V	HST	[145]
1991 March 3	0.70	Ι	HST	[145]
1992 April	0.67	Ι	Hiltner	[181]
1992 April	0.69	V	Hiltner	[181]
1992 April	0.72	Ι	CTIO	[181]
1992 April	0.68	V	CTIO	[181]
1993 April	0.69	Ι	Hiltner	[181]
1993 April	0.63	V	Hiltner	[181]
1995 December 20	0.66	V	Magellan	[162]
1996 June 7	0.68	Ι	NOT	[146]
1997 November 17	0.64	Н	HST	[155]
-	0.52	V	HST	[182]
-	0.67	Ι	HST	[182]
-	0.63	Н	HST	[182]
2001 March 26	0.66	V	Magellan	[179]
2001 April 20–27	0.74	V	1.5-m (Maidanak)	This work
2001 April 20-27	0.67	R	1.5-m (Maidanak)	This work
2001 April 20-27	0.72	Ι	1.5-m (Maidanak)	This work
2002 March	0.76	V	1.5-m (Maidanak)	This work

**Table 3.3.** Estimates of the A2/A1 brightness ratios in PG 1115+030 for the time period 1980–2006 from all available data.

2002 March	0.71	R	1.5-m (Maidanak)	This work
2002 March	0.72	Ι	1.5-m (Maidanak)	This work
2004 February 22	0.81	Sloan i'	Magellan	[179]
2004 May 5–6	0.93	11.67 μm	Subaru	[183]
2004 January 17–June 8	0.79	V	1.5-m Maidanak	This work
2004 January 17–June 8	0.81	R	1.5-m Maidanak	This work
2004 April 11–June 8	0.83	Ι	1.5-m Maidanak	This work
2005 June 7	0.81	Sloan i'	Magellan	[179]
2006 January 5–April 15	0.80	V	1.5-m Maidanak	This work
2006 January 5–June 2	0.83	R	1.5-m Maidanak	This work
2006 January 5–April 15	0.85	Ι	1.5-m Maidanak	This work

In our Table 3.3, the telescopes and filter bands are indicated for all estimates of the A2/A1 flux ratios. The table does not contain the results of the *Chandra* X-rays observations, which exhibited strong flux ratio anomaly and can be found in e.g. [162,179,180]. The data in Table 3.3 are also supplemented by the estimates of the A2/A1 flux ratios obtained from our photometry. Flux ratios in filters V and I from Table 3.3 are displayed in Fig. 3.5. These flux ratios behave similarly in time for both filters, and in general features resemble fig. 2 from [179]. Based upon their fig. 2, authors of [179] argue that the optical flux ratio anomaly in PG 1115+080 is slight and 'nearly constant in time'. Our analysis described below have shown, however, that it is not quite so.

Our fig. 3.5, where the available previous data in V and I are supplemented by our measurements, shows that variations of the A2/A1 flux ratio in time are indeed rather small and slow. However, even if we exclude a marginal value for the date 1983 March 8 [144], which equals 1 with the uncertainty of 0.1, we will have the A2/A1 flux ratio varying in some regular manner with the amplitude of about 0.15 during the last 25 yr. Somewhere between 1991 and 1996, the ratio reached its minimal value of about 0.65 in filter V, and increased up to 0.8 by 2006. It should be noted that the fact that A2/A1 flux ratio varies in time is in itself an argument in favour of microlensing as the main reason for the anomalous flux ratio in PG 1115+080. Furthermore, it should be mentioned that the A2/A1 flux ratio is slightly but steadily higher in filter I than in V.



**Fig. 3.5.** A history of the *A*2/*A*1 flux ratios in PG 1115+080 from the data in filters V and I as listed in Table 3.3.

To determine which component (or components) exactly underwent microlensing, we addressed only our data as more homogeneous ones, and analysed behaviours in time of the long-term constituents of the A2–A1, C–A1, B–C and C–A2 magnitude differences for filters R and I. These difference light curves in filter R are shown in Fig. 3.6.

We did not correct the individual light curves for the time delays, which are small as compared to the characteristic time-scale of quasar flux variations. This might result only in some increase of the data points scatter with respect to the approximating curves, which are the second-order polynomials in Fig. 3.6.

The largest decrease of the magnitude difference is for the A2–A1 image pair – about 0.23 mag during 2001–2005. Pair C–A1 shows an almost linear decrease of the magnitude difference in time, with only 0.12 mag during 2001– 2005. Since the mutual brightness of images B and C was almost invariable in 2001–2006, one might conclude that it is an image A1 that became fainter during this time period. However, the C–A2 magnitude difference curve shows that, in addition to the obvious dimming of image A1, brightening of image A2 makes a certain contribution to the decrease of the A2–A1 magnitude difference in 2001–2005.



**Fig. 3.6.** Behaviours of the C-A1, C-A2, B-C and A2-A1 magnitude differences in time from the results of our photometry in filter R; approximation by the second-order polynomials is shown.

Therefore, we can conclude that a decay of A1 and brightening of A2 took place simultaneously in PG 1115+080 during 2001–2006. We may also conclude that it is the A1 image that underwent microlensing in the previous years, with the maximum near 1992–1995, as seen from Fig. 3.5, and the final phase in 2006 or, perhaps, later. With the previous data taken into account (see Fig. 3.5 and Table 3.3), the total time-scale of the 0.3-mag event is about 25 yr. Image A2 underwent microlensing as well, with its rising branch occurring in 2001–2005. The brightening of image A2 reached about 0.14 mag during this time period, while the total brightening in the whole event may be larger. In calculation of the time

delays, more subtle variations of the magnitude differences during 2004–2006 were found.

It should be noted that our results are well consistent with measurements of the A1–A2 magnitude difference presented in [182], who reported approximately 0.2-mag growth of this quantity during 2001–2006. However, they do not present the magnitude differences between other images and A1 or A2 separately, which has led us to a conclusion about the final phase of microlensing in image A1 and, seemingly, the initial phase of a microlensing event in image A2. This conclusion is also indirectly confirmed by the results of [179], who reported a dramatic rise in the X-ray flux from image A2 between 2001 and 2008. Larger microlensing amplitudes at shorter wavelengths are often detected for many lensed quasars and are known to be naturally explained by smaller effective sizes of quasars at shorter wavelengths.

The observed time-scales and amplitudes of the microlensing brightness fluctuations are known to depend on the relative velocity of a quasar and lensing galaxy, and on the relationship between the source size and the Einstein ring radius of a microlens. For PG 1115+080, the expected duration of a microlensing event is estimated to be of the order of 10 to 20 yr for the subsolar mass microlens [183], well consistent with that in image A1 observed in 1980–2006.

**Table 3.4.** Flux ratios in PG 1115+080 as predicted by the most recent lens models and determined from the results of our photometry in 2006 (filter I); the uncertainty of our flux ratio estimates is 0.02 for all ratios.

Lens model	A2/A1	B/A1	C/A1	B/C
[159]	0.92	0.22	0.28	0.8
[183]	0.92	0.22	0.28	0.79
[162]	0.96		0.26	0.67
[180]	0.92	0.21	0.27	0.78
This work	0.85	0.19	0.29	0.68

Thus, if our interpretation of the observed brightness variations in the A1, A2, B and C images is valid, then the A2/A1 flux ratio would be expected to approach its undisturbed value in a few years, unless a new event takes place in at least one image. Our estimates of the A2/A1, B/A1, C/A1 and B/C flux ratios calculated from the photometry data of 2006 are presented in Table 3.4 along with the model predictions by [159,183] and [162,180]. The ratios for filter I are presented, where the effect of microlensing is expected to be minimal as compared to V and R. As is seen from Table 3.4, the A1/A2 flux ratio is still less than predicted by the most recent lens models. However, when expressed in terms of  $R_{\text{fold}}$ , it would equal 0.08, which means that, according to simulations of [176], this flux ratio is within a region admissible by a smooth lensing potential model for the finite source distances from the caustic fold, i.e. it is not anomalous in the sense implied in [174].

Let us consider colour changes in PG1115+080 now. We have also made use of our multicolour observations to analyse behaviours of the V – I colour indices of image components, which were shown to be indicative of the microlensing nature of brightness changes for at least the Q2237+0305 quasar [151]. Since only 16 data points were available to build the V – I versus *R* dependency for each image component, we did not build them for the components separately, but combined the data into two sets, for A1+A2 and B+C image pairs.

The resulting diagrams are presented in Fig. 3.7. In order to eliminate the magnitude difference and possible permanent colour difference between the components in each pair, we shifted the data points in both plots along the R-axis by the values of the mean magnitude differences between the components. The values of V - I grew with the growth of the R magnitude in both diagrams, with the regression line slopes of 0.28–0.29. This is qualitatively consistent with the total 0.4-mag fading of the PG 1115+080 quasar during 2001–2005: according to numerous observations, there is a common tendency for many quasars to become bluer at their bright phases (see e.g. [184] and references therein). In particular,

authors of [184] presented the  $\Delta(B - R)$  versus  $\Delta B$  and  $\Delta(B - R)$  versus  $\Delta R$  diagrams built for a subset of 21 quasars from their Palomar Green sample consisted of 42 quasars. Their diagrams show a significant correlation between the colour and magnitude variations, with the regression line slopes of 0.25–0.27.



**Fig. 3.7.** V - I versus R diagrams for image A (upper panel) and B, C (bottom), illustrating a statistical dependence between the colour indices and magnitudes. The regression line slopes differ insignificantly for the A1, A2 and B, C image pairs and equal 0.29 and 0.28, with the correlation indices 0.61 and 0.56, respectively.

The regression line slopes for the diagrams in our Fig. 3.7 are also close to that reported for Q2237+0305 in one of our previous publications [151]. There is an important difference between the two quadruple lenses, however: the Q2237+0305 light curves are strongly dominated by microlensing events, while in PG1115+080, a contribution from microlensing activity is small as compared to the quasar intrinsic variability. We do not see any significant difference between the two diagrams in fig. 3.7, though the contributions from microlensing for these image pairs are different. Therefore, the diagrams in Fig. 3.7 should be referred to

characteristics of the PG 1115+080 quasar variability rather than to microlensing variability, in contrast to Q2237+0305, where they result mostly from microlensing.

Furthermore, we tried to analyse the long-term history of colour changes in PG 1115+080 and plotted the values of V – I averaged within every season as functions of the corresponding time moments, which are the mid-points of the seasons (see fig. 3.8). It should be remembered that the values of colour indices in this plot are accurate to only  $\sim \pm 0.025$  mag, therefore, fig. 3.8 illustrates their behaviours in time qualitatively rather than quantitatively. Nevertheless, though the data points in this plot are rather scattered and often overlap giving a rather intricate pattern, the general features in behaviours of colour indices are evident.



**Fig. 3.8.** A history of the long-term variations of colour indices V–I in PG1115+080. Each point is a result of averaging of V–I values within the corresponding season; the mid-points of every season are along the horizontal axis.

To make them clearer, we showed the corresponding linear approximations in Fig. 3.8. First we notice that the V - I colour indices increase in time for all the four image components. This conclusion seems to be valid, while the differences in

gradients of the V – I colours with time are hardly veridical. In general, Fig. 3.8 confirms the dominating contribution from the quasar intrinsic variability over microlensing activity in PG 1115+080 light curves, as was indicated in the section above. Another significant conclusion from Fig. 3.8 is that the V – I colour index of image B remains permanently smaller that those of the other three ones, while demonstrating similar increase as well. We fail to explain why an image that lies the smallest distance from the lensing galaxy turned out to have the bluest colour index.

#### §3.4. Estimation of the Hubble constant

As is noted in Introduction, the time delay between one of the image pairs, say,  $\Delta t_{BC}$ , can be used to determine  $H_0$ , while the time delay ratio  $r_{ABC} = \Delta t_{AC}/\Delta t_{BA}$  is independent of the  $H_0$  value and can be used to constrain the lens model. Most of the PG 1115+080 macromodels are consistent in predicting  $r_{ABC}$  to within 0.15. As far as can be expected from Table 3.3, the three measurements of time delays do not provide the time delay ratios consistent with each other and with model predictions. This can be seen from Table 3.2, where we collected several model predictions for  $r_{ABC}$  together with the measurements presented in [2,59,62]. Furthermore, we presented here the time delay ratios  $r_{CBA} = \Delta t_{AC}/\Delta t_{BC}$ , which are connected with  $r_{ABC}$  by a relationship  $r_{CBA} = r_{ABC}/(1 + r_{ABC})$ . These quantities demonstrate better agreement between model predictions and measurements.

The time delay ratios calculated from measurements of [59] and [62] are seen to be lower as compared to the model predictions, while our measurements provide the estimates of this quantities exceeding the model predictions, especially for  $r_{ABC} = \Delta t_{AC} / \Delta t_{BA}$ , which is as large as 2.73 from our data. One should admit that such discrepancy is too large, since the largest  $r_{ABC}$  we have found in the literature is that calculated in [62] for their isothermal ellipsoid model:  $r_{ABC} = 1.8$ . The reason for this becomes qualitatively clear when addressing the data in Table 3.1: the shortest time delay  $\Delta t_{BA}$  is measured with almost the same absolute error as the longest one,  $\Delta t_{BC}$ , i.e. of all the three time delays,  $\Delta t_{BA}$  has the largest relative error. Therefore, we are far from arguing our value of  $\Delta t_{BA}$  to be more trustworthy that those obtained by [59] and [62]. We regard, however, that the values of  $\Delta t_{BC}$  and  $\Delta t_{AC}$  are more reliable and trustworthy. Furthermore, it should be noted that none of the macrolens models we could found in the literature predicts the values of  $\tau_{BC}$  larger than 18 d – the value 19.9 d from the unrealistic point-mass model in [62] may hardly be taken into account.

**Table 3.5.** Time delays as predicted by the lens models calculated by Schechter et al. (1997) and the values of the Hubble constant  $H_0$  obtained from comparison of these time delays with those obtained by Schechter et al. (1997), columns 2–4; the H0 values calculated for the same lens models with the time delays determined in this work (columns 5–7).

M - 1-1	Lens models, $\Delta t$ (d) and $H_0$ from [62]		$H_0$ with $\Delta t_{AC} = 12.0$ d and $\Delta t_{BC} = 16.4$ d from our work [4]			
Model	$ au_{\rm AC}$ (d)	$ au_{\rm BC}$ (d)	$H_0$	$H_0(\tau_{\rm AC})$	$H_0(\tau_{\rm BC})$	$H_0$ (mean)
PMXS	12.5	19.9	84	104	121	113
ISXS	6.6	10.4	44	55	63	59
ISEP	9.7	15.1	64	81	92	86
ISIS	5.6	9.7	41	47	59	53
ISIS+	5.7	10	42	48	61	54

In the framework of the present publication, we did not intend to either propose an extra exotic lens model or recalculate the most popular ones to derive the new estimate of the Hubble constant, but instead, we have made use of the results of authors of [62], who calculated five models for the gravitational potential of PG 1115+080. The time delays  $\Delta t_{AC}$  and  $\Delta t_{BC}$  as predicted by their five models for  $\Omega = 1$  and  $H_0 = 100$  km s<sup>-1</sup> Mpc<sup>-1</sup>, and the corresponding estimates of  $H_0$ obtained with their time delays are shown in Table 3.5 (columns 2–4). We remind that, according to Schechter et al. (1997), model PMXS means a point mass with external shear, the ISXS model is an isothermal sphere with external shear and ISEP is an isothermal elliptical potential. The ISIS model uses a second isothermal sphere for a group of galaxies at approximately the same redshift as the main lensing galaxy [138,143] to represent shear. In the ISIS+ model, the uncertainty in the galaxy position is not regarded to be negligible, as in the ISXS and ISIS, but its coordinates were taken as two additional free parameters.

Columns 5–7 of Table 3.5 contain the  $H_0$  values estimated for the Schechter et al. models with our values of the time delays for image pairs AC and BC separately (columns 5 and 6), and the average between the pairs (column 7). We may conclude that, as could be expected, our new estimates of  $\Delta t_{BC}$  and  $\Delta t_{AC}$ provide higher values of Hubble constant, which are closer to the most recent value obtained in the *HST* Key Project from observations of Cepheids [185].

# §3.5. Optical observations and microlensing in GLQ B1422+231

Another example of quadruply imaged quasar is B1422+231. Authors of [186] have performed optical and radio observations of a far quasar with redshift  $z_q = 3.62$  and was first who identified it as quadruple GLQ with brightness of 16.5 mag. It is one of bright lensed systems. Infrared observations of B1422+231 was led authors of [187] in the same year where authors have made attempt to definition of redshift the lensing galaxy  $z_g \sim 1$ . Authors of [188] confirmed the previous conclusions on the basis of optical observations and identified 4 point sources and magnitudes. It has turned out, that brightness of A, B, C, D components in the V filter are 16.71 mag, 16.46 mag, 17.26 mag and 20.42 mag accordingly. They have not found presence of the fifth component. In 1994 [189] and [190] modeled B1422+231. In the first case one galaxy with an isothermal sphere and two galaxies as point sources were considered as a lens model. And in the second case it was used isothermal ellipsoid. Both these models well reproduce gravitational system B1422+231.

Authors of [191] again tried to receive exact parameters of the lensing galaxy in the B1422+231. Photometric properties of the system and the mass/brightness ratio of this galaxy allow to make assumption that, probably, it is a galaxy of early type with redshift  $z_g = [0.4 - 0.8]$ . In 1995 [192] received new value of  $z_g = 0.647$ . Authors of [193] presented relative fluxes in V, R and I filters for all a component of system B1422. Brightness of the system as a whole in R filter was  $m_{A+B+C+D} = 15.59 \pm 0.06$  mag and flux ratios of components were A = 1.0, B = 1.52, C = 0.73, D = 0.037. [194] for the first time assumed presence of microlensing in B1422. The group [195] informed about short periodic variability of quasar B1422 during one day. Authors of [196] informed that the flux ratios between components in optical and radio bands essentially differ. Photometric properties of the lensing galaxy correspond to an elliptic galaxy with redshift  $z_{g} \sim 0.4$ . Later there was found that the lens galaxy belongs to compact cluster with  $z_{cl} = 0.338$ , radius  $R_{cl} = 35 \cdot h^{-1}$  and mass  $M = 1.4 \pm 10^{13} \cdot h^{-1}$  [197]. These results have been confirmed in [140].

Authors of [198] presented the flux ratios between components in several radio-bans. Significant discrepancy between flux ratios of A component in optical, infra-red and radio bands they explained by presence of microlensing. Authors of the paper [199] have carried out detailed research of conditions of excitation, ionization and a chemical mixture using spectroscopy in near infra-red band. Then [200] presented polarization observations of the gravitational lens system B1422+231 made at 8.4GHz using the VLBA and the 100-m telescope at Effelsberg.

In result of them were the detailed card of this system is received and surface brightness of A and B components is investigated. In 2001 on the basis of radio observations the value of time delay between components of B1422 for the first time have been published [201]. Using the cross correlation analysis they have shown that time delay between pairs is  $\Delta t_{BA} = 1.5 \pm 1.4$  day,  $\Delta t_{AC} = 7.6 \pm$ 2.5 days and  $\Delta t_{BC} = 8.2 \pm 2.0$  days. Authors of [202] using the new observant data had modeled the lens galaxy. They have found out that mass substructure of the galaxies plays the important role in strong lensing in case B1422. Authors of [203] with help Chandra observations had detected X-ray radiation in the galaxies cluster around B1422. The time delay between components A and C  $\Delta t \sim 0.4 h^{-1}$ has been predicted. Grant et al. in 2004 also using X-ray images of the Chandra observatory investigated properties of the galaxies cluster associated with the system B1422 [204]. And last work on B1422 is the paper of [183], where it is informed on infrared observations and new limitations on a substructure of the lens galaxy. In this work we presented the results of three years optical monitoring of B1422 in Maidanak observatory.



**Fig. 3.9.** R-filter image of B1422+231 obtained on April 13 2004. The multiple B1422+231. Clear visible the three quasar images, the reference stars S1, S2 components A, B and C. Our detector and the PSF star) are illustrated. The field can not register the D component. The size is  $\sim 2.1$  arcmin  $\times 1.83$  arcmin.

3.10. Zooming Fig. image of field size is  $\sim 8 \times 8$  arcsec.

In this section we present our multi-colour observant data of GLS B1422 (Fig. 3.9) obtained in Maidanak observatory [81]. These observations were carried out during the periods from May, 3 till August, 14, 2003, from January, 17 till August, 13, 2004 and at last from April, 1 till June, 29, 2005. We shall note that in

the specified periods we tried to observe this object with the period 2-3 days as far as weather allowed us. Observations were carried out with CCD detector BROCAM (2000 x 800 pix.) with pixel scale 0.26 arcsec and the size of a field of 8.7 arcmin  $\times$  3.5 arcmin. This detector is equipped by Johnson-Cousins filters system. By us have been used V and R filters. On the average we observed on 4 frames per night with an exposition of 150 seconds in R and 180 seconds in V the filter. The median of quality of the sky is 1.07. Thus, we have received 132 nights in R the filter and 26 nights in V the filter. We have rejected images with bad quality of the image (more, than 1.5 at FWHM) and high value of the sky background(it is more, than 1500 counts). After that we still had 92 nights in R filter and 20 nights in V the filter.

Photometry of the B1422 is not easy, because its all components too close one to another where maximum separation is 1.3 arcsec (Fig. 3.10). Therefore we can not use the DAOPHOT package [87]. At the same time, because of the timevariable and slightly asymmetric PSF of AZT-22 it was not possible to use the image subtraction method [123,124]. For photometry of the B1422 components we chose the GALFIT program [205]. The quoted error bars are the standard 1- $\sigma$ errors determined by GALFIT. For construction of the point spread function model we took the star marked "PSF" (Fig. 3.9). Normalization of PSF model was performed by using standard package IRAF. In our processing we not took account the presence of the lensing galaxy because it is too faint to be presence in our frames ( $m_{R,g} = 21.8 \text{ mag}$ ,  $m_{R,g} = 21.8 \text{ mag}$ , [191,195]). Besaide it we ignored and D component also by the same reasons ( $m_{R,g} = 20.42 \text{ mag}$ ). Because "PSF" star is quietly bright star it was chosen as reference star. During observation period this star was stable. In Fig.3.11 the light curve of "PSF" relatively "S1' star shown. The reference stars was processed by aperture photometry.



**Fig. 3.11.** The light curves of the quasar B1422+231 and the reference star S2 in R-band during 2003-2008

As we can see in the Fig.3.11 the source-quasar shows noticeable variation of the brightness due to its internal variability. At the very beginning of the observation campaign, in 2003, the shine of the quasar was at its peak. From the next season, brightness of the system immediately rushed down to its "quasi-quiet" state, and the fall continued until the end of 2006. And from next year it is clear that the brightness began to increase again. The amplitude of the variation is ~ 0.25 mag. This remarkable behavior of total brightness of the source-quasar can be explained, apparently, by accretion of a matter to the center of the quasar, which causes an increase in brightness.



**Fig. 3.12.** The light curve of the components A, B and C of the quasar B1422+231 in the R-band. The light curve of the reference star S2 is shown also. In the future we used an updated version of Galfit, i.e. in the input parameters, the components' coordinates could be rigidly related to the central one. As a result we obtained the following graphs, shown in Fig. 3.12. In general, the brightness of the components vary according to the brightness of the source as a whole. At least two microlensing events can be indicated – in 2003 and in the last of 2008. In both cases, the light curves of the components B and C as a whole coincide, but the behavior of component A differs markedly from them.



**Fig. 3.13.** V - R colour curve of B1422+231 for the quasar images A (red), B (green), C (blue) and reference star (shifted for clarity, magenta).

Though on our light curves the microlensing obviously it is not shown, nevertheless it can be defined indirectly. As in the case of microlensing occurs not only temporal changing of brightness of components, but also change of their colours. Therefore for such analysis it is possible to plot a curve of an colour index, which is shown in fig 3.13. Here it is visible that colour indexes of components B and C between 2004 and 2005 years have a trend and were a little shifted to the blue side. But the colour of A component has changed. This fact can be the indirect certificate of microlensing in this GLQ.



 $\Delta t_{AB} = 2.5 \pm 2.3$  days  $\Delta t_{AC} = -7 \pm 1.5$  days  $\Delta t_{BC} = 6 \pm 1.8$  days **Fig. 3.14.** *Top*: Observed and interpolated light curves light curves; *bottom*: histograms of the most likely time delays between 3 pairs of lensed components in B1422+231.

Taking as a basis the obtained light curves of the lensed components in B1422 + 231 we tried to measure the delay time between variations in their brightness. A well-known polynomial fitting and interpolation method was used to analyze the light curves and measure the delay time, which is well suited for uneven and discontinuous data. With the help of polynomials, we obtained an artificial uniform series. And then, using the Monte Carlo method with random numbers, we obtained 1000 random realizations of the light curves for both objects (Fig. 6). The complexity of the polynomial fitting method is the optimal choice of the polynomial degree. It is necessary to describe the observed brightness

variability as accurately as possible, i.e. not to suppress the natural variability (both long-period and short-period). And at the same time, interpolation should not give additional, often false signs of variability.

At the same time, we used to interpolate the entire light curve as a whole, not the seasons separately. Thus, we were able to find the time delay values based on the long-period changes that occur in the source-quasar. In Fig.3.14. the observed light curves and their interpolated analogues (thin curves) are given. The histograms below show the delay time values for random realizations of the light curves. Also the mean time delays and confidence intervals are shown.

# §3.6. Summary and discussion on Chapter III

From our new data of excellent quality (Tables 6–8), we had a possibility to discriminate between the quasar intrinsic and microlensing brightness and colour variations for PG 1115+080, and to obtain new estimates of the time delays. It may immediately be seen from our R light curves, especially for 2004–2006 seasons, that signifi-cant brightness fluctuations with amplitudes exceeding the typical error bars of the data points have been detected. This allows us to recompute the values of  $H_0$  calculated with the previous estimates of the time delays for this gravitational lens system.

In particular, we find the following: we have studied behaviours of brightness ratios of all the components both in time and in wavelength. We report microlensing in A1 with an amplitude of about 0.3 mag in filter R on a time-scale of 25 yr. The magnification peak in A1 took place in 1992–1995, with the subsequent fading in 2001–2006. The image A1 flux may be expected to reach its undisturbed value by 2006 or later. A microlensing event was apparently observed in image A2 as well, with its rising branch in 2001–2005, when image A2 brightened by approximately 0.15 mag. The time-scales and amplitudes of both events are consistent with those predicted for this object for the solar mass

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microlenses [183]. In fitting the 2004–2006 data points to the approximating function, very subtle signs of microlensing have also been found in image B.

Therefore, deviation of the observed A2/A1 flux ratio from that predicted by most of the lens models can be well explained by microlensing events. An additional contribution to the flux ratio 'anomaly' may be expected from the source position with respect to the caustic fold: when expressed in terms of Rfold (equation 2), the brightness difference between A1 and A2 would equal 0.08, which means that, according to simulations in [176], it is within a region admissible by a smooth lensing potential model, i.e. it is not anomalous in the sense implied first in [174].

We have made use of observations in other filters available for some dates to analyse behaviours of colour indices of the images. The V – I versus R diagrams built for pairs A1+A2 and B+C demonstrate the known tendency of quasars to become bluer at their bright phases, with no signs of any contribution from microlensing: the diagrams for both image pairs are nearly identical. This can be explained either by poor statistics (the data in all the three filters are available for only 16 nights, providing rather poor correlation as indicated in Fig. 3.7 caption), or by small amplitudes of the microlensing events under consideration, or by both reasons

An interesting feature of the behaviours of colour indices should be noted. While all images demonstrate growth of their colour indices in time, the V – I colour indices of image B are slightly but steadily less than those of other images. This is rather unexpected, since image B is located the closest distance to the main lensing galaxy.

The time delays for PG 1115+080 obtained from our monitoring data in 2004–2006 differ from those determined in [59] and [62] earlier. The differences for  $\tau_{BA}$  and  $\tau_{BC}$  are well beyond the uncertainties reported in both publications and determined in the present work. While our time delay estimates for images A and
C are rather close to the two previous ones, the delays for two other image pairs cannot be regarded as consistent even marginally.

As could be expected, our estimates of time delays  $\Delta t_{AC}$  and  $\Delta t_{BC}$  result in larger  $H_0$  values than those reported in [62] with their estimates of time delays and with the ISXS, ISIS and ISIS+ models again in [62]. The new estimates of  $H_0$  are more consistent with the most recent  $H_0$  value obtained in the HST Key Project [185].

The new estimates of time delays in PG 1115+080 provide additional support for the family of models close to isothermal. As analysed in details in [164], the estimates of  $H_0$  with the use of time-delay lenses are bounded by two limiting models: models with less dark matter (more centrally concentrated mass profiles) produce higher values of  $H_0$  than those with more dark matter. In particular, the constant mass-to-light ratio models set an upper limit on estimates of  $H_0$ , while the isothermal mass distribution models are responsible for the lower limit of  $H_0$ . Our result is very important in this respect, since an isothermal model is preferred for the lensing galaxy in PG 1115+080 for the reasons listed in [165]: (1) the velocity dispersions observed for an ensemble of lensing galaxies are consistent with the fundamental plane relations for ellipticals; (2) a majority of the nearby galaxies, as well as those lensing galaxies for which the radial mass distributions can be measured, are very nearly isothermal. Since the PG 1115+080 lensing galaxy is by no means unusual, the isothermal hypothesis is most probable.

In conclusion, observations of PG 115+080 by Morgan et al. 2008 in filter R during almost exactly the same time periods in 2004–2006 should be mentioned. We have used their table 3 photometry to compare to our light curves. The quasar brightness fluctuations which allowed us to determine the time delays are seen in their A1+A2 light curve quite well, but become undetectable in the B and C light curves because of a much larger scatter of the data points.

We present V -filter and R-filter photometry of the gravitational lens system B1422 over 3 years - 2003 (May to August), 2004 (January – August) and 2005

(April to June). We have observed brightness variations of the quasar with amplitude ~  $0.1^{m}$  and a time scale of the variations of several years. Gaps (due to the observing conditions at the telescope) and small photometric accuracy (due to difficult structure of images) in our light curves make a determination of the time-delay between images impossible. Though microlensing effect on our light curves obviously it is not visible, nevertheless with the help of changes of an colour index of components we progress to receive indirect indication of microlensing.

Using the polynomial interpolation method, we were able to calculate new values for time delays in B1422+231. The maximum value does not exceed 1 week. The large spread of values does not yet allow for more accurate values of this parameter.

# CHAPTER IV. NEW PHOTOMETRIC PROCESSING METHOD AND LIGHT CURVES OF THE GLQ H1413+117: TIME DELAYS AND MICRO-LENSING EFFECTS

## §4.1. On observations and research of the H1413+117

As we mentioned above gravitational lensing offers a unique tool to study the mass distribution of the lens, the structure of the light source as well as the geometry of the Universe. However, any significant contribution to these endeavours requires some continuous and detailed investigations of each known gravitational lens system. One example of such a long-term investigation of a gravitationally lensed quasar is H1413+117, also known as the Clover Leaf, whose spectrum was first observed in [206] and [207]. Later it became evident that this object is a lensed quasar displaying four images [208]. This source is relatively distant ( $z_s \approx 2.55$ ) and its brightness classifies it as a highly luminous quasar ( $M_V < -29$  and  $m_V \approx 17$ ), while its spectrum revealed it to be a broad absorption line quasar [206]. The four lensed images display an almost symmetric configuration, the maximum angular separation between them being 1.36 arcsec. But the nature of the lensing body still remains open.

On the basis of the comparison between optical and radio observations, two potential gravitational lens models of the Clover Leaf have been proposed: (a) a single elliptical galaxy or (b) two spherical galaxies. Theoretical estimations of the time delays have led to values of about several weeks [209]. The first light curves of the individual lensed images of H1413+117 in the Bessel V filter have been reported in [210]. They very sparsely cover the time interval between 1987 and 1993. The light curves of the individual components varied almost simultaneously, which indicates that the time delays should be much shorter than the sampling period ( $\sim$ 1 yr). Subsequent re-analysis [211] of the same data led to some more elaborated conclusions. Namely, (a) the fluxes of the A, B, C components vary in parallel and almost simultaneously; (b) the expected time delays between the

lensed components must be shorter than 1 month. On the other side, the weakest D component showed additional light variations besides the global trend, which was the first evidence reported for a micro-lensing event in the H1413+117 system [211].

In a series of papers [212-214], detailed studies of the optical images and spectra of the lensed quasar H1413+117 obtained with the HST and Multiple Mirror Telescope were carried out. A faint elongated arc between the A and C components seems to indicate the presence of an Einstein ring-like structure [213]. According to this paper, any future model of this system must take into account the different colours of the components, i.e. the line-of-sight-dependent extinction. In addition to three metal-line systems of absorption lines at  $z_{abs} = 1.66, 1.44, 2.07$  [206,208], numerous absorption lines were found at  $z_{abs} = 0.61 \div 2.1$  [212,214]. The lensing galaxy had not been detected, but its redshift was expected to be  $z_{lens} > 1$ . Otherwise, the lens would have been detected on the HST images [212, 214]. Using astrometric and photometric data obtained in [213,215] constrained at best plausible lens models for the Clover Leaf. Another group showed that an external shear is needed to correctly model this system [147].

Authors of [216] found new pieces of evidence for micro-lensing of the D component and showed that this component significantly displays different relative polarization properties compared to the three other ones. More recently, additional numerous spectroscopic pieces of evidence of micro-lensing were reported for the components of the Clover Leaf. Most micro-lensing activity is seen for the D component, which variability lasts for at least several years [217,218]. Moreover, the micro-lensing of the D component shows a chromatic dependence [219]. This result is directly consistent with the estimation of the duration of the micro-lensing events in H1413+117 in [220].

The lensing body of H1413+117 represents another difficulty. During a long time, this has only been investigated theoretically. The first observational conclusions about this deflector were provided in [221,222]. They estimated the

brightness of the lensing galaxy to be  $m_{160W} \approx 20.6$ , and concluded that it may belong to a galaxy cluster with a redshift  $z_{cluster} \approx 1.7$ . These results were later supported in [223], who confirmed that the lensing galaxy is a really diffuse and faint object:  $m_{160W} \approx 20.5$  and  $m_{180W} \approx 22.2$ . Those conclusions were enriched by the precise astrometry of the lensed components of the system via mid-infrared and modeling investigations in [224].

Finally, we should note that the only work dedicated to the observational determination of the time delays between the components of the Clover Leaf has been performed in [225]. Using r Sloan band observations in 2008, they were able to predict the values of the time delays between three pairs of components:  $\Delta_{AB} = -17 \pm 3$  days,  $\Delta_{AC} = -20 \pm 4$  days,  $\Delta_{AD} = 23 \pm 4$  days (where A, B and C are leading, and D is the trailing component). Moreover, those authors, using their new time delay constraints, improved the lens model and estimated the possible redshift of the lensing galaxy:  $z_{lens} = 1.88^{+0.09}_{-0.11}$ . In this chapter, we present the results of long-term monitoring observations of H1413+117 carried out at the Maidanak observatory between 2001 and 2008 (in the R and V bands).

The observational data of H1413+117 (see Fig. 4.1 and Fig. 4.2) were collected between 2001 and 2008 during a quasar monitoring programme carried out by the Maidanak Gravitational Lens System collaboration. CCD frames in the Bessel R and V bands were obtained with the 1.5-m AZT-22 telescope of the Maidanak Observatory using two different CCD cameras [81,226]. During all seasons except 2007, the frames were obtained with the 2000 × 800 pixel SITe-005 CCD camera. Among them, the CCD frames of 2001 and 2002 were taken in the long-focus mode, while the other ones were taken in the short-focus mode. The angular pixel sizes are 0.135 and 0.267 arcsec, respectively. CCD frames in 2007 were taken with the 4096 × 4096 SNUCAM (provided by Seoul University CAMera) camera in the short-focus mode, which has a pixel scale of 0.267 arcsec and a field of view of  $18.1 \times 18.1$  arcmin. The photometric filters used were the R and V filters of the Johnson–Cousins filter system. We observed on average six

frames per night, with an exposure time of 180 s in R and 210 s in V. A summary of the observational data acquired between 2001 and 2008 is given in Table 4.1.

Table 4.1. Summary of the H1413+117 Clover Leaf observational data collect	ed
during the seasons from 2001 to 2008 (under good observational conditions).	

Seasons	R-band		V-band	
	Period	NN*	Period	NN
2001	21 May	1	21 May	1
2002	3 March – 9 May	6	26 March – 9 April	2
2003	28 January – 31 July	20	7 May – 19 July	10
2004	22 January – 4 August	32	26 February – 4 August	22
2005	8 February – 30 June	20	13 July	1
2006	4 January – 18 July	20		
2007	25 May – 17 July	4		
2008	9 May – 4 June	5		
Danial air	ton in dd/man			

Period given in dd/mm.

\*NN – number of nights.



**Fig. 4.1.** R-band CCD frame of the GLS H1413+117 obtained with the Maidanak 1.5-m telescope, on 2003 February 10. The quasar and the reference stars R1, R2 and R3 are labelled. The field angular size is  $2.2 \times 2.2$  arcmin. The stars R1 and R2 are named as Objects 40 and 45 in table 2 in [209].

Our data cover eight observational seasons and consist of 202 CCD frames in the R-filter and 54 CCD frames in the V filter (each of these frames is the result of an average combination of the frames which were obtained during the given night). However, a careful analysis of the data forced us to reject frames obtained under bad observing conditions: a seeing > 1.2 arcsec and a sky background value  $> 2000 \text{ ADU pixel}^{-1}$ .



**Fig. 4.2.** Zoomed R-band images of the central parts of H1413+117. Left: long-focus mode (pixel size is 0.135 arcsec; 2001 May 21). The seeing is ~ 0.73 arcsec and the image size is  $1.35 \times 1.35$  arcsec. Right: short-focus mode (pixel size is 0.265 arcsec; 2003 February 10). The seeing is ~0.66 arcsec and the image size is  $26.5 \times 26.5$  arcsec. Orientation: North is down, East to the right.

Nevertheless, in some cases of excellent seeing, we accepted the frames affected by a sky background value as large as 4200 ADU pixel<sup>-1</sup>. Finally, we are left with 108 frames in the R filter and 35 frames in the V filter. The average seeing and signal-to-noise ratio measured for a field star R1 having a brightness similar to those of the quasar images are 1.03 arcsec and 131.8, respectively. These data are given in Table 4.2. The pre-treatment applied to the Maidanak data consisting of the standard procedure (bias correction and flat-fielding) was done using exclusively the standard IRAF software. The results of the photometry of the lensed images and their analysis are presented in the next sections.

## §4.2. The adaptive PSF-fitting photometric method

Several methods have been developed to measure the flux of multiple sources with a small separation (e.g. [114,227,228,229]). The majority of these methods requires the assumption of PSF invariance over the frame. This postulate imposes some restrictions on the photometric methods, as the observing conditions or the imaging system do not always encounter this assumption. We propose a new approach to overcome this limitation.

The adaptive PSF-fitting method is an efficient photometric technique that aims at determining the PSF from the lensed components themselves. The fluxes of the images are needed to accurately determine the PSF, but they also need the PSF to be accurately calculated. Consequently, this method constitutes an iterative process in order to calculate both of them.

Let us consider a CCD frame consisting of  $M \times N$  pixels. The flux distribution of the source is represented by D(i, j), and the average value of the sky background by  $S_b$ . The corresponding pixel flux uncertainty is then defined as  $\sigma(i, j)$ :

$$\sigma(\mathbf{i},\mathbf{j}) = \frac{\sqrt{([D(\mathbf{i},\mathbf{j}) + S_b)] * gn + rd^2)}}{gn}$$

where rd is the read-out noise  $(e^-)$  and  $gn(e^-ADU^-)$  the gain of the CCD. Here  $i \in (1, ..., M)$  and  $j \in (1, ..., N)$ . Let us assume that a number of K point-like sources contribute to D(i, j). In our case, these point-like sources correspond to the four lensed quasar (QSO) images of H1413+117. Each lensed component may be represented by the unknown PSF(i,j) centred at the position  $(i_k, j_k)$  with a central intensity  $I_k$ :

$$D(i,j) = \sum_{k}^{K} I_{k} * PSF(i - i_{k}, j - j_{k}) + \varepsilon(i,j), \qquad (4.1)$$

where  $\varepsilon(i, j)$  represents the noise contribution. The adaptive PSF fitting method consists in reducing the  $\chi^2$  between D(i, j) and the sum of all the scaled PSFs by adjusting the values of the I<sub>k</sub> and PSF(i, j) parameters until reaching convergence. This  $\chi^2$  function is defined as:

$$\chi^{2} = \sum_{i=1,j=1}^{M,N} \frac{\left[\sum_{k=1}^{K} I_{k} * PSF(i - i_{k}, j - j_{k}) - D(i, j)\right]^{2}}{\sigma^{2}(i, j)}.$$
 (4.2)

To determine both sets of parameters, it is necessary to alternatively calculate them: adopting initial values for the first set of parameters, the second set can be calculated by minimizing the  $\chi^2$ . Then this second set is used to re-calculate the first set of parameters, which will be re-used to determine the values of the second one, etc. Since we are attempting to analyse a frame in order to extract some flux information, it is simpler to provide a relevant initial PSF than providing relevant initial central intensities. Let us assume that we have a first known set of PSF(i, j), we have then to minimize the  $\chi^2$  in order to determine the intensities  $I_k$ . It can be done by equating the partial derivatives  $\partial \chi^2 / \partial I_l$  to zero:

$$\frac{1}{2} \frac{\partial \chi^2}{\partial I_l} = \sum_{i,j=1}^{M,N} \left[ PSF(i-i_l,j-j_l) \times \frac{\sum_{k=1}^{K} I_k * PSF(i-i_k,j-j_k) - D(i,j)}{\sigma^2(i,j)} \right] = 0$$

$$(4.3)$$

This relation can be simplified as:

$$\sum_{k=1}^{K} \sum_{i,j=1}^{N,M} \frac{\text{PSF}(i - i_l, j - j_l) \cdot \text{PSF}(i - i_k, j - j_k)}{\sigma^2(i, j)} \cdot I_k =$$

$$= \sum_{i,j=1}^{N,M} \frac{D(i, j) \cdot \text{PSF}(i - i_l, j - j_l)}{\sigma^2(i, j)}$$
(4.4)

which leads to

$$\sum_{k=1}^{K} M_{l,k} \cdot I_k = C_l$$
 (4.5)

with

$$M_{l,k} = \sum_{i,j=1}^{N,M} \frac{\text{PSF}(i - i_l, j - j_l) \cdot \text{PSF}(i - i_k, j - j_k)}{\sigma^2(i,j)}$$
(4.6)

and

$$C_{l} = \sum_{i,j=1}^{N,M} D(i,j) \cdot \frac{\text{PSF}(i-i_{l},j-j_{l})}{\sigma^{2}(i,j)}$$
(4.7)

Equation (4.5) can be rewritten in a vectorial form:

$$M \cdot \vec{I} = \vec{C} \quad \text{or} \quad \vec{I} = M^{-1} \cdot \vec{C} \tag{4.8}$$

and it is easy to derive the components of the vector  $\vec{I}(I_1, ..., I_k)$  if we assume that we know what are the values of the PSF. We thus proceed in an iterative way.

Suppose that the PSF, centred on (0,0) covers  $I \times J$  pixels, where  $I = 2M_l + 1$  and  $J = 2M_J + 1$ . In the remainder, we use two different coordinate systems in order to locate one pixel on such a grid:  $i' \in [-M_I, M_I]$ ,  $j' \in [-M_J, M_J]$  and the pixel number  $n = i' + (j' - 1)(2M_I + 1) \in [1, I \times J]$ . So, for a given pair of coordinates (i', j') we obtain a unique value of n, and vice versa.

Considering any value for the number  $n \in [1, I \times J]$  specific to a given PSF element, we know that only *K* values of D(i, j) will be concerned over the whole frame. As written before, a value of *n* defines a pair of values (i', j'). Knowing that there is a number *K* of PSFs, we should consider each of these independently. Therefore, for a value of  $l \in [1, K]$  and a given pair of coordinates (i', j'), it is straightforward to find the absolute value  $(i_{nl}, j_{nl})$  of the position of the PSF(l) in the frame. If  $(i_l, j_l)$  represent the absolute central position of the PSF(l), then  $i_{nl} = i_l + i'$  and  $j_{nl} = j_l + j'$ .

We can now perform the derivation of the  $\chi^2$  function given in equation (4.2) with respect to the chosen PSF(*n*). We find

$$\frac{\partial \chi^2}{\partial \text{PSF}(n)} = 2 \sum_{l=1}^{K} \left[ \frac{\sum_{k\neq l}^{K} I_k \cdot \text{PSF}(i_{nl} - i_k, j_{nl} - j_k)}{\sigma^2(i_{nl}, j_{nl})} + \frac{I_l \cdot \text{PSF}(n) - D(i_{nl}, j_{nl})}{\sigma^2(i_{nl}, j_{nl})} \right] \cdot I_l = 0$$

$$(4.9)$$

and thus,

$$PSF(n) \sum_{l=1}^{K} \frac{I_l^2}{\sigma^2(i_{nl}, j_{nl})} = \\ = -\sum_{l=1}^{K} \left[ \frac{\sum_{k\neq l}^{K} I_k \cdot PSF(i_{nl} - i_k, j_{nl} - j_k) - D(i_{nl}, j_{nl})}{\sigma^2(i_{nl}, j_{nl})} \right] \cdot I_l \quad (4.10)$$

The updated value of PSF(n) = PSF(i', j') can be recalculated as PSF(n)

$$PSF(n) = \frac{\sum_{l=1}^{K} \left[ \frac{D(i_{nl}, j_{nl}) - \sum_{k \neq l}^{K} I_k \cdot PSF(i_{nl} - i_k, j_{nl} - j_k)}{\sigma^2(i_{nl}, j_{nl})} \right] \cdot I_l}{\sum_{l=1}^{K} \frac{I_l^2}{\sigma^2(i_{nl}, j_{nl})}}, \quad (4.11)$$

with the new intensities  $I_l$  being solutions of equation (4.8).

At the first iteration, the input point spread function  $PSF(i - i_k, j - j_k)$  can be any peaked 2D function or the image of some nearby reference star. It is then used as the input PSF(n) in equation (4.8) to derive a set of new central intensities. The latter are then injected in equation (4.11) to derive new values of PSF(n) and so on. After a series of iterations, we derive the best values of PSF(n) and of the central intensities which minimize the  $\chi^2$ .

As we see, the two equations (4.8) and (4.11) constitute the core of the adaptive PSF-fitting method. By using them alternatively and starting from a first estimation of the PSF, the central intensities  $I_k$  can be numerically calculated, as well as the PSF. The adaptive PSF-fitting method has a major advantage over the classical PSF-fitting method: there is no need to make use of any assumption concerning the invariance of the PSF over the CCD frame. This calculated PSF is determined from the lensed components themselves, not from a reference star. We have performed the tests with different shapes of the initial PSF and found that the result does not depend on it. As the iterations proceed, the PSF always converges towards a stable final shape which is typical for a given observation. Indeed, in equation (4.1) we only made the assumption that the PSFs of each component are identical to each other. This assumption of the local frame invariance is only

effective over the distance scale of the gravitational lens system. This approximation is much less restrictive than the global frame invariance assumed in the classical PSF-fitting method.



**Fig. 4.3.** a) real PSF; b) composed image with two real PSFs including Poisson noise; c) initial PSF; d) derived PSF after a series of iterations; e) residuals between the derived and the real PSF, normalized to one standard deviation  $(1\sigma)$  of the expected Poisson noise; f) residuals after subtraction of a double PSF, also normalized to  $1\sigma$ .

The only requirements specific to the adaptive PSF-fitting method are:

(i) the positions of each component on the CCD frame have to be accurately known, at least within a pixel size precision;

(ii) the CCD frames must have a good signal-to-noise ratio. It is then possible in our case to pinpoint the four overlapping components of H1413+117.

Let us note that these requirements are shared by most of photometric techniques dedicated to the flux measurement of very nearby objects. Thus, the absence of additional assumptions implies that the adaptive PSF-fitting photometric method can be applied to a larger range of observations than any other photometric technique.

We tested our method using artificially generated images. First of all, we chose some 2D PSF (see Fig. 4.3a). Secondly, we composed a CCD frame which is the sum of two real PSFs with a flux ratio and a separation between their centres equal to 0.7 and 5 px (pixels), respectively. Moreover, we added random Poisson noise to this frame (Fig. 4.3b). For the first step of the adaptive PSF fitting, we chose some initial PSF to solve equations (4.5)–(4.8) (Fig. 4.3c). We carried out a series of iterations until the total  $\chi^2$  reached a given minimum stable value. As a result, we derived a PSF (Fig. 4.3d) which is very similar to the real one.

The residuals between the derived and the real PSF are shown in Fig. 4.3(e) after normalization to  $\sigma(i, j)$ , one standard deviation of the expected Poisson noise. As can be seen, there is no significant difference between the two, thus we have correctly recovered the primary, real PSF.

Next, we checked the results of the method for different flux ratios and distances between the centres of the components. First, we fixed the separation between the two components to Dist = 5px and applied the adaptive PSF-fitting method for three different values of the flux ratio  $A_1/A_2 = 0.1, 0.5, 0.7$ . For each fixed value of the flux ratio and distance, we randomly generated 15 frames affected by Poisson noise (Fig. 4.3b), and then applied to each of these the adaptive PSF fitting.

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**Fig. 4.4.** Examples of the flux ratio and  $\chi^2$  value evolutions as a function of the number of iterations for different present flux ratios and separations. Top row: the separation between the two components is Dist = 5 px everywhere, the flux ratios  $A_1/A_2 = 0.1, 0.5, 0.7$ . Bottom row: flux ratios  $A_1/A_2 = 0.7$  everywhere, the separations are D = 1, 3, 5 px.

Fig. 4.4 (top row) shows the evolution of the flux ratios between the components as a function of the iteration step for different values of the flux ratio  $A_1/A_2$ . As can be seen, the curves converge towards the adopted value after just a few iterations. Average values of the flux ratios are  $0.100 \pm 0.001$ ,  $0.500 \pm 0.002$ ,  $0.7 \pm 0.003$ , respectively. Secondly, we fixed the flux ratio between the two components to 0.7 and applied the adaptive PSF-fitting method for three different values of their separation: 1, 2, 5 px. Obviously, wider is the pair, better is the final accuracy. The bottom row of Fig. 4.4 shows how the flux ratio converges as a function of the number of iterations. Only small changes occur after the 10 first iterations, with the calculated flux ratios:  $0.730 \pm 0.091$ ,  $0.720 \pm 0.075$ ,  $0.700 \pm 0.002$ .

## §4.3. Photometry of the four lensed components of H1413+117 and light

#### curves

We shall now describe the application of the adaptive PSF-fitting photometric method to the case of the Clover Leaf. Since most of our CCD frames were obtained in the short-focus mode, we shall focus our attention to the problems encountered with this type of CCD frames.

Our method does not require a high accuracy on the lensed component positions – it is enough to know them within a single pixel size precision. To determine the relative distances between the components we used all available information about the coordinates of the lensed images. We adopted the astrometric measurements provided by [213,223,224] as a first approximation to fix the component positions which were registered on our detector. Inaccuracies in the positions of the components were taken into account when estimating the photometric error bars (see Table 4.2).

However, application of the adaptive PSF-fitting photometric method to the original frames did not give any satisfactory results, first of all, due to the small signal-to-noise ratio. On the other hand, H1413+117 is a multiple object with small angular separations between the lensed components. The combination of these two factors did not allow us to distinguish (even under good seeing conditions) the separate peaks of the lensed images. In this case, the adaptive PSF-fitting photometric method led to non-physical results – negative fluxes or implausible flux ratios of the components. In general, the mathematical model of the adaptive PSF-fitting method allows us to reach such solutions. To avoid such difficulties, we have decided to artificially increase the resolution of the images using a bi-cubic interpolation of the original frames [230]. The details of the interpolation are explained in [231] and were also applied in [232, 233].

An average sky background value is simply subtracted before applying the adaptive PSF-fitting photometric method (this is not implicitly included in equations (4.8) and (4.11)). A typical result calculated by means of the adaptive

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PSF-fitting photometric method on to a bi-cubic interpolated CCD frame is shown in Fig. 4.5.



**Fig. 4.5.** Results of the application of the adaptive PSF-fitting photometric method to our CCD frames. (a) The original frame, obtained in the R band on 2003 February 10 (the same as in Fig. 4.2 at right); (b) 10-times pixelated and bi-cubic interpolated version of the original frame; (c) the resulting adaptive PSF-fitting photometric model of H1413+117; (d) the residual image between the real frame and the simulated one. This frame is given in units of the standard noise deviation  $\sigma(i, j)$ . The presented scale and overall distribution of the residuals are characteristic of all the considered CCD frames.

Here are the original frame (a) and a frame which was pixelated 10 times (b), which means that each pixel of the original CCD frame was bi-cubic

interpolated from the 10 inner points equidistant from one another. The frame (c) illustrates the resulting adaptive PSF photometric model; and in (d) the residuals between the real and modelled frames, normalized to the one sigma standard noise deviation  $\sigma(i,j)$  image frame. The residual frames show good general self-consistency; their individual values are in the range of  $\pm (2-3) \sigma(i,j)$ . The criterion used for stopping the iteration was to reach a stable convergence for the flux ratios of the lensed components. In the case of good seeing conditions, about 1200–1600 iterations were needed to reach reliable results. For all images of the Clover Leaf, we used the image of the star R1 as the initial PSF, observed on 2004 May 09 under good conditions.

At first, we would like to present the instrumental photometric light curves of H1413+117 as a whole and those of the reference star R2 in the V and the R bands, as well as the V-R colour curves (see Fig. 4.6).



**Fig. 4.6.** Right: light curves of the quasar H1413+117 (blue and red points) and of the reference star R2 (violet and dark-blue) in the *V*- and *R*- pass band, respectively. For clarity, both light curves of R2 are shifted down by 0.25 mag. Left: V-R colour curves of the quasar and of the reference star R2. Quasar light curves are plotted relative to the reference star R2, and R2 are plotted relative to the star R3.



**Fig. 4.7.** R-band light curves for the A, B, C, D lensed components of H1413+117 and of the reference star R1, which has been shifted up by -0.5 mag for clarity, with respect to the reference star R2. The years which correspond to the observing seasons are indicated at the top.



**Fig. 4.8.** *V*-band light curves for the A, B, C, D lensed components of H1413+117 and of the reference star R1, which has been shifted up by -0.75 mag for clarity, with respect to the reference star R2. The years which correspond to the observing seasons are indicated at the top.

The light curves were obtained via aperture photometry using the DAOPHOT/IRAF software. The light curves of R2 are displayed relatively to the star R3 (see Fig. 4.1) and are shifted down by 0.25 mag for clarity. Given the apparent photometric stability of the star R2 with respect to the star R3, we have decided to use hereafter the star R2 as a reference star. The relative light curve R1–R2 (see Figs 4.7 and 4.8) is also flat and stable, but has greater dispersion (this is due to the fact that the magnitude difference between R1 and R2 (or R3) is around 2 mag).

As seen in Figs 4.6–4.8, the quasar displays some active variability with an amplitude  $\approx 0.15$  mag between the end of the season 2003 and the middle of 2005 (the brightness of the D-lensed component reached its maximum at this moment and then diminished smoothly during the following years). The mentioned activity between the seasons 2003 and 2005 does not show up similarly in the two photometric bands. As we can see, there exists a brightness–colour dependence: a decrease in the brightness of the quasar leads to a reddening, and vice versa. This is an interesting observation and might possibly be explained as due to micro-lensing (as for the case of Q2237+0305, [40]) and/or as internal processes in the quasar itself (as in UM673, [232,234]). To assert this statement, we must separately analyse the light curves and the colour curves of the lensed images.

# §4.4. New method for calculation of the photometric errors

We have applied our adaptive PSF fitting method to the lensed images of H1413+117 to derive their appropriate flux measurements. The next task was then to estimate the photometric error bars affecting each point of the light curves. To calculate these photometric error bars, we used Monte-Carlo simulations of our CCD frames including the different sources of noise. We built simulations of each frame of H1413+117 and applied again our adaptive PSF fitting technique to each of them in order to perform some statistical estimates. The calculated magnitudes

are expected to follow a Gaussian distribution. For this reason, the error bars associated with each point of our light curves are defined as the standard deviation of the simulations corresponding to the observation. Building relevant simulations of photometric observations is not a trivial task and consists of several steps:

<u>Modelling of the Clover Leaf CCD frames.</u> As mentioned above, the program returns the central intensities of each component and a new estimate of the local PSF. PSF fitting of this PSF (using, for example, Gaussian or Moffat functions) in combination with the fluxes and the well-known positions of the lensed components of H1413+117 allows to easily construct simulated frames.

<u>The GLS signal.</u> The flux of the lensed components measured from each original pixel over the CCD is naturally affected by the photon noise. For a given number of N photons falling on a pixel during the entire integration time, the distribution of k measured photons follows the Poisson distribution:

$$P(k,N) = \frac{e^{-N}N^k}{k!}$$

P(k, N) represents the probability of measuring k photons among N received photons. In our case, we use this probability to simulate a series of frames that include the photon noise. This type of distribution can properly describe the outer parts (wings) of starlike objects where we have a relatively small number (up to several dozen) of photons. To describe the noise in the central parts of the simulated objects, we use Gaussian distributions (see below).

<u>GLS, Sky background and read-out noise</u>. The sky background noise also finds its origin in the photon noise. The property of Poisson distributions is to tend towards a normal distribution for large values of N. In the case of a sky background larger than 200 ADU/pixel, it is possible to model the sky background noise with a Gaussian distribution such as:

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma_{bg}} \cdot exp\left(-\frac{1}{2}\frac{\left(x-S_{g}\right)^{2}}{\sigma_{bg}^{2}}\right)$$

$$\sigma_{bg} = \sqrt{\frac{rd_{ef} + S_g}{gn_{eff}}}$$

where  $S_g = b_g + flux_{obj}$  represents then 'effective' signal containing the average value of the sky background  $b_g$  and the flux of the superimposed object  $flux_{obj}$ ,  $\sigma_{bg}$  is the width of the distribution with  $rd_{ef} = rd \cdot \sqrt{n_{fr}}$  and  $gn_{ef} = gn \cdot n_{fr}$ , where  $n_{fr}$  is the number of combined frames for the given day of observation,  $rd_{ef}$ and  $gn_{ef}$  are respectively the effective read-out noise and the corresponding gain of the considered frames [235].

<u>Flat field variation</u>. Each pixel of the CCD has its own sensitivity, independently of its neighbours, except if the CCD was damaged by an external source: an overexposure, a shock, etc. But our CCD frames did not reveal any such artefact. Therefore, the sensitivity of the pixels randomly varies over the entire CCD detector surface. To simulate such variations of the pixel sensitivity over the CCD surface, we have created artificial flat fields, consisting of a pure Gaussian noise frame (with an average value around 1 and with several different widths), and divided our simulated frames by such flat fields.

We tried Gaussian widths equal to 0.005, 0.05 and 0.1 on several series of CCD frames and compared the original and the new sky background distributions. We found that histograms and standard deviations were very slightly affected by the flat field fluctuations. Such variations are negligible, except in the case of a Gaussian width equal to 0.1, but this latter case looks quite unrealistic. Thus, we eventually ignored this source of noise in our models.

<u>Random positional centering</u>. This source of noise has its origin in the random position of the centred object with respect to the pixel grid of the CCD. As can be seen on our frames, the positional centring is correct but its accuracy is limited by

the fractional size of a pixel. Under this scale, nothing guarantees that the target is at the exact centre of a given pixel.

In our models, we took this effect into account as follows: before squeezing the modeled frames, the binning grid was offset by a certain number of sub-pixels along the *X*- and *Y* - directions (by up to 5 sub-pixels in the years 2001 and 2002, and by up to 10 subpixels in 2003 - 2008). This always resulted in a positional shift of the Clover Leaf images by less than 1 pixel.

We can summarise our whole photometric measurement procedure as follows:

- first, each observed frame was pixelated and bi-cubic interpolated in order to resize them. After this procedure every image of the dataset had the same pixel scale;

- the flux of each lensed component and the local *PSF* were calculated using the adaptive PSF fitting method;

- the resulting *PSF* was subsequently fitted using a Moffat model. Simulated images of the Clover Leaf were then built by placing the PSF model at the relative positions of the lensed components and by multiplying them by their central intensities;

**Table 4.2.** Photometry of H1413+117 in the R- and V- bands. Only a portion of the table is shown here for guidance regarding its form and content. The fulltable is available in the online version of the journal (see Supporting Information).

The columns are labelled as follows:

column 1: DD/MM/YY

column 2: Normalized JD = JD - 2450000

column 3: value of the sky background (ADU)

column 4: the number of combined frames for the given observing day

columns 5-6: FWHM of the seeing disc in arcsecond and signal-noise-ratio measured on the base of star R1

columns 7-10: brightness of the A, B, C, D lensed components relative to the R2 reference star (mag) and their photometric errorbars. The method of the calculation of the errorbars will be discussed in Appendix B.

column 11: brightness of the reference star R2 relative to R3 (mag) and its photometric error bars, which were obtained via aperture photometry using the DAOPHOT/IRAF software

	<i>R</i> -band									
Civil Date	JD	S <sub>bg</sub>	N <sub>fr</sub>	FWHM	SNR	A	В	С	D	R2
1	2	3	4	5	6	7	8	9	10	11
21/05/01	2021	52	11	0.90	361	$1.132 \pm 0.004$	$1.260 \pm 0.005$	$1.442 \pm 0.005$	$1.591 \pm 0.005$	$-0.083 \pm 0.002$
03/03/02	2337	174	2	0.94	107	$1.227\pm0.016$	$1.367 \pm 0.017$	$1.468\pm0.018$	$1.688\pm0.021$	$-0.110 \pm 0.004$
05/03/02	2339	74	4	0.91	176	$1.259\pm0.009$	$1.378\pm0.009$	$1.481 \pm 0.009$	$1.673\pm0.010$	$-0.111 \pm 0.003$
08/03/02	2342	55	2	1.10	162	$1.245 \pm 0.019$	$1.344 \pm 0.021$	$1.477 \pm 0.019$	$1.710 \pm 0.023$	$-0.090 \pm 0.003$
14/03/02	2348	44	2	0.97	137	$1.255 \pm 0.012$	$1.411 \pm 0.014$	$1.522 \pm 0.015$	$1.699 \pm 0.017$	INDEF
26/03/02	2360	251	2	0.76	89	$1.214\pm0.011$	$1.367 \pm 0.013$	$1.442\pm0.012$	$1.647\pm0.014$	$-0.075 \pm 0.006$
09/04/02	2374	46	2	0.98	138	$1.226\pm0.009$	$1.382\pm0.010$	$1.495\pm0.010$	$1.672\pm0.012$	$-0.087 \pm 0.003$
27/01/03	2667	300	5	1.03	129	$1.071\pm0.011$	$1.353\pm0.014$	$1.439\pm0.014$	$1.682\pm0.017$	$-0.123 \pm 0.002$
01/02/03	2672	218	5	0.93	152	$1.058\pm0.007$	$1.350\pm0.010$	$1.451 \pm 0.009$	$1.657\pm0.011$	$-0.119 \pm 0.002$
09/02/03	2680	270	7	0.94	162	$1.101 \pm 0.007$	$1.324\pm0.009$	$1.494 \pm 0.009$	$1.659 \pm 0.011$	$-0.116 \pm 0.002$
10/02/03	2681	338	7	0.83	65	$1.105 \pm 0.011$	$1.355 \pm 0.014$	$1.477 \pm 0.015$	$1.644 \pm 0.015$	$-0.127 \pm 0.004$
26/04/03	2756	244	4	1.01	108	$1.105 \pm 0.013$	$1.375 \pm 0.017$	$1.559 \pm 0.017$	$1.662 \pm 0.018$	$-0.119 \pm 0.003$
27/04/03	2757	211	4	0.90	125	$1.143 \pm 0.006$	$1.338\pm0.008$	$1.515 \pm 0.009$	$1.689 \pm 0.010$	$-0.116 \pm 0.002$
05/05/03	2765	243	4	1.00	113	$1.121 \pm 0.010$	$1.370 \pm 0.014$	$1.486 \pm 0.013$	$1.698 \pm 0.016$	$-0.104 \pm 0.002$
07/05/03	2767	284	4	0.79	109	$1.150 \pm 0.006$	$1.342 \pm 0.007$	$1.504 \pm 0.007$	$1.677 \pm 0.009$	$-0.129 \pm 0.001$

17/05/03	2777	1187	4	0.89	44	$1.141 \pm 0.014$	$1.292 \pm 0.017$	$1.536 \pm 0.022$	$1.690 \pm 0.024$	$-0.122 \pm 0.003$
22/05/03	2782	241	4	0.96	94	$1.133 \pm 0.010$	$1.374 \pm 0.013$	$1.532 \pm 0.014$	$1.734 \pm 0.015$	$-0.125 \pm 0.003$
29/05/03	2789	281	4	1.22	108	$1.122 \pm 0.039$	$1.322 \pm 0.055$	$1.503 \pm 0.043$	$1.696 \pm 0.055$	$-0.115 \pm 0.003$
04/06/03	2795	209	4	0.89	114	$1.175 \pm 0.006$	$1.343 \pm 0.007$	$1.519 \pm 0.008$	$1.675 \pm 0.008$	$-0.110 \pm 0.003$
17/06/03	2808	237	4	0.89	99	$1.192 \pm 0.007$	$1.374 \pm 0.008$	$1.572 \pm 0.009$	$1.707 \pm 0.010$	$-0.121 \pm 0.002$
29/06/03	2820	236	4	1.01	107	$1.167 \pm 0.011$	$1.373 \pm 0.013$	$1.538 \pm 0.014$	$1.730 \pm 0.016$	$-0.113 \pm 0.003$
30/06/03	2821	238	4	0.92	108	$1.195\pm0.008$	$1.393 \pm 0.010$	$1.560 \pm 0.010$	$1.692 \pm 0.012$	$-0.124 \pm 0.002$
17/07/03	2838	685	6	1.08	66	$1.202 \pm 0.023$	$1.424 \pm 0.030$	$1.507 \pm 0.028$	$1.699 \pm 0.031$	$-0.110 \pm 0.004$
19/07/03	2840	882	8	1.05	71	$1.188 \pm 0.015$	$1.351 \pm 0.018$	$1.532 \pm 0.019$	$1.709 \pm 0.024$	$-0.112 \pm 0.004$
24/07/03	2845	315	6	1.19	115	$1.196 \pm 0.022$	$1.348 \pm 0.027$	$1.535 \pm 0.029$	$1.744 \pm 0.033$	$-0.114 \pm 0.002$
29/07/03	2850	374	6	1.23	99	$1.229 \pm 0.032$	$1.354 \pm 0.037$	$1.568 \pm 0.036$	$1.680 \pm 0.039$	$-0.123 \pm 0.003$
31/07/03	2852	734	8	1.00	76	$1.210 \pm 0.016$	$1.399 \pm 0.019$	$1.508\pm0.018$	$1.716\pm0.022$	$-0.119 \pm 0.003$
22/01/04	3027	302	4	0.83	178	$1.232 \pm 0.004$	$1.384 \pm 0.005$	$1.555 \pm 0.005$	$1.624\pm0.006$	$-0.125 \pm 0.001$
19/02/04	3055	278	4	1.11	155	$1.225 \pm 0.007$	$1.351 \pm 0.008$	$1.500\pm0.008$	$1.587\pm0.008$	$-0.122 \pm 0.000$
26/02/04	3062	294	3	1.02	122	$1.243\pm0.007$	$1.405\pm0.008$	$1.551 \pm 0.009$	$1.643 \pm 0.009$	$-0.122 \pm 0.002$
27/02/04	3063	278	4	0.74	147	$1.232\pm0.004$	$1.385\pm0.005$	$1.556\pm0.005$	$1.612\pm0.006$	$-0.116 \pm 0.002$
01/03/04	3066	312	4	0.92	157	$1.218\pm0.006$	$1.369 \pm 0.007$	$1.534\pm0.007$	$1.626\pm0.007$	$-0.113 \pm 0.002$
29/03/04	3094	338	6	0.88	182	$1.225\pm0.006$	$1.356\pm0.006$	$1.512\pm0.006$	$1.639\pm0.007$	$-0.117 \pm 0.002$
30/03/04	3095	634	6	1.00	103	$1.233\pm0.010$	$1.354\pm0.012$	$1.528\pm0.013$	$1.676\pm0.016$	$-0.121 \pm 0.003$
10/04/04	3106	379	4	0.81	139	$1.219\pm0.005$	$1.396\pm0.006$	$1.539\pm0.006$	$1.661\pm0.006$	$-0.125 \pm 0.001$
11/04/04	3107	261	6	0.96	191	$1.236\pm0.006$	$1.399\pm0.007$	$1.525 \pm 0.007$	$1.677\pm0.008$	$-0.119 \pm 0.001$
12/04/04	3108	263	6	0.99	203	$1.217\pm0.006$	$1.361\pm0.007$	$1.544\pm0.008$	$1.664\pm0.008$	$-0.120 \pm 0.001$
13/04/04	3109	222	5	0.87	182	$1.244\pm0.005$	$1.399\pm0.006$	$1.560\pm0.007$	$1.691 \pm 0.007$	$-0.099 \pm 0.002$
14/04/04	3110	193	5	0.93	196	$1.229\pm0.006$	$1.422\pm0.007$	$1.557\pm0.007$	$1.703\pm0.008$	$-0.125 \pm 0.001$
23/04/04	3119	230	8	1.21	180	$1.230\pm0.016$	$1.356\pm0.018$	$1.506\pm0.019$	$1.681\pm0.023$	$-0.111 \pm 0.002$
30/04/04	3126	1164	4	1.15	83	$1.237\pm0.025$	$1.346\pm0.030$	$1.495\pm0.028$	$1.650\pm0.034$	$-0.126 \pm 0.004$
04/05/04	3130	233	5	0.96	187	$1.243\pm0.005$	$1.375\pm0.006$	$1.546\pm0.006$	$1.649\pm0.007$	$-0.122 \pm 0.001$
17/05/04	3143	198	6	0.94	154	$1.202\pm0.005$	$1.327\pm0.006$	$1.478\pm0.007$	$1.636\pm0.007$	$-0.116 \pm 0.002$
20/05/04	3146	199	7	1.05	229	$1.231 \pm 0.006$	$1.331 \pm 0.007$	$1.504\pm0.007$	$1.669\pm0.008$	$-0.125 \pm 0.001$
23/05/04	3149	142	10	1.08	206	$1.193\pm0.010$	$1.367 \pm 0.011$	$1.441 \pm 0.010$	$1.652\pm0.012$	$-0.115 \pm 0.002$
26/05/04	3152	276	7	0.91	121	$1.180\pm0.007$	$1.321 \pm 0.009$	$1.480 \pm 0.009$	$1.653 \pm 0.011$	$-0.124 \pm 0.002$
08/06/04	3165	265	4	0.88	154	$1.169 \pm 0.005$	$1.325 \pm 0.006$	$1.492 \pm 0.006$	$1.639 \pm 0.006$	$-0.123 \pm 0.002$
10/06/04	3167	212	4	1.13	162	$1.175 \pm 0.013$	$1.371 \pm 0.015$	$1.468\pm0.014$	$1.670 \pm 0.017$	$-0.125 \pm 0.002$
11/06/04	3168	209	4	0.98	156	$1.170 \pm 0.006$	$1.315 \pm 0.007$	$1.455 \pm 0.007$	$1.631 \pm 0.008$	$-0.124 \pm 0.002$
19/06/04	3176	233	4	1.08	126	$1.167 \pm 0.010$	$1.316 \pm 0.011$	$1.451 \pm 0.012$	$1.621 \pm 0.014$	$-0.129 \pm 0.002$
21/06/04	3178	213	4	1.07	139	$1.170\pm0.009$	$1.309\pm0.010$	$1.481\pm0.011$	$1.594\pm0.011$	$-0.123 \pm 0.002$
23/06/04	3180	324	3	1.00	104	$1.128\pm0.010$	$1.328\pm0.013$	$1.532\pm0.013$	$1.591\pm0.013$	$-0.122 \pm 0.002$

29/06/04	3186	2852	6	0.97	53	$1.120\pm0.016$	$1.268 \pm 0.019$	$1.503 \pm 0.020$	$1.533\pm0.021$	$-0.117 \pm 0.005$
08/07/04	3195	253	8	0.97	207	$1.180\pm0.005$	$1.285 \pm 0.006$	$1.459 \pm 0.006$	$1.519\pm0.006$	$-0.118 \pm 0.001$
10/07/04	3197	426	7	0.99	151	$1.174 \pm 0.008$	$1.312 \pm 0.010$	$1.486 \pm 0.011$	$1.589 \pm 0.012$	$-0.120 \pm 0.002$
22/07/04	3209	381	8	1.13	163	$1.165 \pm 0.010$	$1.294 \pm 0.012$	$1.449 \pm 0.013$	$1.528\pm0.014$	$-0.113 \pm 0.002$
24/07/04	3211	1408	6	0.96	73	$1.160 \pm 0.013$	$1.308 \pm 0.016$	$1.473 \pm 0.017$	$1.509 \pm 0.017$	$-0.126 \pm 0.005$
03/08/04	3221	286	6	1.09	133	$1.111 \pm 0.011$	$1.309 \pm 0.013$	$1.443 \pm 0.014$	$1.468 \pm 0.015$	$-0.118 \pm 0.003$
04/08/04	3222	419	6	1.07	114	$1.142 \pm 0.012$	$1.292 \pm 0.014$	$1.463 \pm 0.015$	$1.477 \pm 0.014$	$-0.141 \pm 0.003$
08/02/05	3410	258	6	0.83	189	$1.144\pm0.004$	$1.278 \pm 0.005$	$1.477 \pm 0.006$	$1.538\pm0.006$	$-0.142 \pm 0.001$
13/02/05	3415	240	6	1.04	186	$1.153\pm0.008$	$1.207 \pm 0.009$	$1.481 \pm 0.010$	$1.507\pm0.010$	$-0.131 \pm 0.002$
20/02/05	3422	89	8	0.89	119	$1.114 \pm 0.007$	$1.208 \pm 0.008$	$1.456 \pm 0.009$	$1.507 \pm 0.009$	$-0.137 \pm 0.002$
01/03/05	3431	1191	8	1.14	102	$1.123\pm0.021$	$1.201 \pm 0.022$	$1.446 \pm 0.026$	$1.511 \pm 0.026$	$-0.145 \pm 0.003$
04/03/05	3434	237	6	0.92	192	$1.161 \pm 0.005$	$1.256 \pm 0.006$	$1.481 \pm 0.006$	$1.493 \pm 0.007$	$-0.148 \pm 0.002$
20/03/05	3450	258	4	1.06	136	$1.133\pm0.011$	$1.209 \pm 0.012$	$1.435 \pm 0.013$	$1.515\pm0.014$	$-0.141 \pm 0.002$
30/03/05	3460	1016	7	0.87	68	$1.138\pm0.010$	$1.226\pm0.010$	$1.438\pm0.012$	$1.481\pm0.012$	$-0.147 \pm 0.003$
14/04/05	3475	237	8	0.83	211	$1.135\pm0.003$	$1.225 \pm 0.004$	$1.446\pm0.005$	$1.497\pm0.005$	$-0.152 \pm 0.001$
16/04/05	3477	269	8	0.88	186	$1.116\pm0.004$	$1.248\pm0.005$	$1.450\pm0.006$	$1.507\pm0.006$	$-0.148 \pm 0.001$
27/04/05	3488	1287	7	0.94	85	$1.106\pm0.009$	$1.209\pm0.010$	$1.433\pm0.012$	$1.468\pm0.012$	$-0.149 \pm 0.003$
09/05/05	3500	215	7	1.09	188	$1.099\pm0.008$	$1.210 \pm 0.009$	$1.432\pm0.011$	$1.528\pm0.012$	$-0.150 \pm 0.002$
10/05/05	3501	245	6	1.11	111	$1.067\pm0.010$	$1.202\pm0.012$	$1.436\pm0.013$	$1.456\pm0.013$	$-0.147 \pm 0.003$
15/05/05	3506	351	8	1.20	186	$1.096\pm0.016$	$1.217\pm0.019$	$1.418\pm0.018$	$1.507\pm0.020$	$-0.154 \pm 0.002$
16/05/05	3507	451	8	1.02	174	$1.080\pm0.007$	$1.224\pm0.008$	$1.446\pm0.009$	$1.489\pm0.009$	$-0.156 \pm 0.002$
25/05/05	3516	1422	8	1.06	81	$1.105\pm0.018$	$1.165\pm0.020$	$1.402\pm0.021$	$1.476\pm0.022$	$-0.141 \pm 0.003$
03/06/05	3525	247	6	0.96	170	$1.162\pm0.006$	$1.281\pm0.007$	$1.482\pm0.007$	$1.520\pm0.007$	$-0.120 \pm 0.002$
14/06/05	3536	491	4	1.01	108	$1.140\pm0.010$	$1.289\pm0.012$	$1.506\pm0.014$	$1.508\pm0.014$	$-0.122 \pm 0.003$
20/06/05	3542	2563	3	0.92	44	$1.122\pm0.021$	$1.189\pm0.023$	$1.485\pm0.027$	$1.492\pm0.027$	$-0.113 \pm 0.007$
26/06/05	3548	342	4	1.00	89	$1.165\pm0.011$	$1.251\pm0.012$	$1.454\pm0.014$	$1.466\pm0.013$	$-0.128 \pm 0.003$
30/06/05	3552	213	4	1.16	134	$1.165\pm0.012$	$1.206\pm0.014$	$1.466\pm0.015$	$1.489\pm0.016$	$-0.125 \pm 0.002$
04/01/06	3740	401	10	1.04	281	$1.144\pm0.005$	$1.272\pm0.006$	$1.483\pm0.007$	$1.682\pm0.008$	$-0.130 \pm 0.001$
27/02/06	3794	400	8	1.05	246	$1.171\pm0.006$	$1.277\pm0.007$	$1.495\pm0.008$	$1.695\pm0.010$	$-0.137 \pm 0.001$
22/03/06	3817	454	8	0.97	253	$1.177\pm0.006$	$1.292\pm0.007$	$1.497\pm0.007$	$1.680\pm0.009$	$-0.121 \pm 0.001$
01/04/06	3827	297	7	1.19	266	$1.195\pm0.007$	$1.297\pm0.008$	$1.470\pm0.009$	$1.679\pm0.011$	$-0.115 \pm 0.001$
11/04/06	3837	3587	8	1.12	99	$1.169\pm0.021$	$1.274 \pm 0.025$	$1.453 \pm 0.024$	$1.669\pm0.032$	$-0.124 \pm 0.003$
17/04/06	3843	1172	8	0.91	168	$1.176\pm0.006$	$1.269 \pm 0.007$	$1.499 \pm 0.008$	$1.620\pm0.008$	$-0.122 \pm 0.002$
25/04/06	3851	256	7	0.80	204	$1.203\pm0.004$	$1.294\pm0.004$	$1.508\pm0.005$	$1.668\pm0.005$	$-0.130 \pm 0.001$
10/05/06	3866	4254	8	1.00	78	$1.124\pm0.012$	$1.225\pm0.012$	$1.470\pm0.013$	$1.658\pm0.017$	$-0.124 \pm 0.003$
15/05/06	3871	2551	8	0.83	54	$1.150\pm0.010$	$1.267 \pm 0.011$	$1.459\pm0.013$	$1.598\pm0.014$	$-0.103 \pm 0.005$
18/05/06	3874	392	6	1.42	192	$1.137\pm0.022$	$1.306\pm0.027$	$1.465\pm0.027$	$1.630\pm0.030$	$-0.123 \pm 0.001$

20/05/06	3876	304	8	1.29	192	$1.165\pm0.017$	$1.239 \pm 0.019$	$1.485\pm0.020$	$1.686\pm0.026$	$-0.127 \pm 0.002$
24/05/06	3880	349	8	1.20	223	$1.192\pm0.010$	$1.280\pm0.011$	$1.524\pm0.012$	$1.647\pm0.015$	$-0.127 \pm 0.001$
27/05/06	3883	316	8	1.40	263	$1.156\pm0.013$	$1.260 \pm 0.015$	$1.467\pm0.016$	$1.660 \pm 0.021$	$-0.136 \pm 0.001$
03/06/06	3890	595	8	1.30	177	$1.108\pm0.015$	$1.259\pm0.018$	$1.475 \pm 0.019$	$1.666 \pm 0.024$	$-0.129 \pm 0.002$
14/06/06	3901	1162	8	1.19	139	$1.135\pm0.016$	$1.266 \pm 0.020$	$1.443\pm0.021$	$1.681 \pm 0.025$	$-0.124 \pm 0.002$
18/06/06	3905	320	8	1.12	248	$1.169\pm0.008$	$1.278 \pm 0.009$	$1.532 \pm 0.011$	$1.659 \pm 0.012$	$-0.130 \pm 0.001$
22/06/06	3909	329	8	1.35	234	$1.134\pm0.019$	$1.319 \pm 0.023$	$1.443\pm0.022$	$1.690 \pm 0.028$	$-0.129 \pm 0.001$
29/06/06	3916	431	8	1.21	206	$1.143\pm0.012$	$1.308\pm0.015$	$1.506\pm0.016$	$1.677\pm0.018$	$-0.127 \pm 0.002$
14/07/06	3931	733	8	1.40	160	$1.167\pm0.027$	$1.270 \pm 0.030$	$1.462\pm0.029$	$1.673\pm0.037$	$-0.132 \pm 0.002$
18/07/06	3935	373	8	1.43	192	$1.156\pm0.027$	$1.293 \pm 0.030$	$1.456\pm0.030$	$1.671 \pm 0.039$	$-0.130 \pm 0.002$
25/04/07	4210	494	3	1.00	242	$1.206\pm0.005$	$1.434\pm0.006$	$1.581\pm0.007$	$1.814\pm0.008$	$-0.114 \pm 0.001$
13/05/07	4228	290	5	1.23	233	$1.172\pm0.013$	$1.409\pm0.017$	$1.535\pm0.016$	$1.783\pm0.021$	$-0.126 \pm 0.001$
10/05/07	4286	293	4	1.04	167	$1.214\pm0.007$	$1.395 \pm 0.009$	$1.566\pm0.010$	$1.803\pm0.012$	$-0.111 \pm 0.001$
17/05/07	4293	301	3	1.18	169	$1.175\pm0.017$	$1.377 \pm 0.020$	$1.488\pm0.018$	$1.798\pm0.024$	$-0.116 \pm 0.001$
09/05/08	4596	193	5	0.90	143	$1.181\pm0.006$	$1.360\pm0.008$	$1.563\pm0.008$	$1.742\pm0.010$	$-0.128 \pm 0.002$
13/05/08	4600	176	5	1.43	134	$1.153\pm0.030$	$1.321\pm0.038$	$1.527\pm0.037$	$1.703\pm0.047$	$-0.121 \pm 0.002$
27/05/08	4614	197	3	1.02	135	$1.173\pm0.008$	$1.354\pm0.009$	$1.555\pm0.010$	$1.741\pm0.012$	$-0.114 \pm 0.002$
31/05/08	4618	257	3	1.15	95	$1.\;175\pm 0.018$	$1.350\pm0.024$	$1.535\pm0.023$	$1.712\pm0.027$	$-0.123 \pm 0.003$
0.1/0.1/0.0	4(2)	202	2	0.94	107	$1.202 \pm 0.005$	$1.200 \pm 0.006$	$1.550 \pm 0.007$	$1.742 \pm 0.009$	$0.147 \pm 0.002$
04/06/08	4622	202	3	0.84	127	$1.203 \pm 0.005$	$1.590 \pm 0.000$	$1.539 \pm 0.007$	$1.742 \pm 0.008$	$-0.147 \pm 0.002$
04/06/08	4622	202	3	0.84	127	$\frac{1.203 \pm 0.005}{V \text{ bane}}$	$1.390 \pm 0.000$	1.559 ± 0.007	$1.742 \pm 0.008$	$-0.147 \pm 0.002$
04/06/08 Civil Date	JD	<i>S<sub>bg</sub></i>	N <sub>fr</sub>	FWHM	SNR	$\frac{1.203 \pm 0.003}{V \text{ bank}}$	$1.390 \pm 0.000$	1.559 ± 0.007	D	-0.147 ± 0.002
Civil Date 1	JD 2	<i>S<sub>bg</sub></i>	5 N <sub>fr</sub> 4	5 0.84	SNR 6	$\frac{1.203 \pm 0.003}{V \text{ bane}}$	$\frac{1.390 \pm 0.000}{B}$	C 9	D 10	-0.147 ± 0.002
O4/06/08           Civil           Date           1           21/04/21	<b>JD</b> 2 2021	<b>S</b> <sub>bg</sub> <b>3</b> 26	N         N           4         5	0.84 FWHM 5 1.02	<b>SNR</b> <b>6</b> 144	$     1.203 \pm 0.003 \\     V bane     A     7     1.000 \pm 0.007     $	$1.390 \pm 0.000$ <b>B</b> <b>B</b> <b>1.173 \pm 0.008</b>	c 1.339 $\pm 0.007$ 9 1.311 $\pm 0.008$	<i>D</i> 1.499±0.011	-0.147 ± 0.002 <b>R2</b> 11 0.076±0.008
O4/06/08           Civil           Date           1           21/04/21           26/03/02	<b>JD</b> 2021 2360	<b>S</b> <sub>bg</sub> <b>3</b> 26 223	Nfr           4           5           2	<b>FWHM</b> <b>5</b> 1.02 0.90	<b>SNR</b> <b>6</b> 144 45		$B$ $1.173\pm0.008$ $1.224\pm0.023$	$\begin{array}{c} \textbf{C} \\ \textbf{9} \\ 1.311 \pm 0.008 \\ 1.331 \pm 0.023 \end{array}$	<i>D</i> 1.499±0.011 1.583±0.022	$\begin{array}{c} \textbf{R2} \\ \hline \textbf{11} \\ 0.076 \pm 0.008 \\ 0.077 \pm 0.017 \end{array}$
O4/06/08           Civil           Date           1           21/04/21           26/03/02           09/04/02	<b>JD</b> 2021 2360 2374	<b>S</b> <sub>bg</sub> <b>3</b> 26 223 23	N <sub>fr</sub> 4           5           2           2	<b>FWHM</b> <b>5</b> 1.02 0.90 1.00	<b>SNR</b> <b>6</b> 144 45 79		$B$ $B$ $1.173\pm0.008$ $1.224\pm0.023$ $1.341\pm0.016$	$\begin{array}{c} \textbf{C} \\ \textbf{9} \\ \hline 1.311 \pm 0.008 \\ \hline 1.331 \pm 0.023 \\ \hline 1.360 \pm 0.017 \end{array}$	D           10           1.499±0.011           1.583±0.022           1.569±0.019	$\begin{array}{c} \textbf{R2} \\ \hline \textbf{R2} \\ \hline \textbf{0.076} \pm 0.008 \\ \hline 0.077 \pm 0.017 \\ \hline 0.110 \pm 0.011 \end{array}$
O4/06/08           Civil           Date           1           21/04/21           26/03/02           09/04/02           07/05/03	JD           2           2021           2360           2374           2767	202 <b>S</b> <sub>bg</sub> <b>3</b> 26 223 23 136	Nfr           4           5           2           2           4	0.84 FWHM 5 1.02 0.90 1.00 0.86	<b>SNR</b> <b>6</b> 144 45 79 77	$\begin{array}{r} 1.203 \pm 0.003 \\ \hline V \text{ bane} \\ \hline A \\ \hline 7 \\ 1.000 \pm 0.007 \\ \hline 1.113 \pm 0.020 \\ \hline 1.108 \pm 0.012 \\ \hline 1.047 \pm 0.035 \end{array}$	$\begin{array}{r} 1.390 \pm 0.006 \\ \hline \textbf{B} \\ \hline \textbf{B} \\ \hline \textbf{B} \\ \hline 1.173 \pm 0.008 \\ \hline 1.224 \pm 0.023 \\ \hline 1.341 \pm 0.016 \\ \hline 1.308 \pm 0.116 \end{array}$	$\begin{array}{c} \textbf{C} \\ \textbf{9} \\ \hline 1.311 \pm 0.008 \\ \hline 1.331 \pm 0.023 \\ \hline 1.360 \pm 0.017 \\ \hline 1.385 \pm 0.031 \end{array}$	D           10           1.499±0.011           1.583±0.022           1.569±0.019           1.591±0.077	$\begin{array}{c} \textbf{R2} \\ \hline \textbf{11} \\ 0.076 \pm 0.008 \\ 0.077 \pm 0.017 \\ 0.110 \pm 0.011 \\ 0.035 \pm 0.011 \end{array}$
O4/06/08           Civil           Date           1           21/04/21           26/03/02           09/04/02           07/05/03           29/05/03	JD           2           2021           2360           2374           2767           2789	202 <b>S</b> <sub>bg</sub> 26 223 23 136 143	N <sub>fr</sub> 4           5           2           4           4	0.84           FWHM           5           1.02           0.90           1.00           0.86           1.13	<b>SNR</b> <b>6</b> 144 45 79 77 66	$\begin{array}{r} 1.203 \pm 0.003 \\\hline V \text{ bane} \\\hline A \\\hline 7 \\\hline 1.000 \pm 0.007 \\\hline 1.113 \pm 0.020 \\\hline 1.108 \pm 0.012 \\\hline 1.047 \pm 0.035 \\\hline 1.043 \pm 0.023 \\\hline \end{array}$	$\begin{array}{r} 1.390 \pm 0.006 \\ \hline \textbf{B} \\ \hline \textbf{B} \\ \hline \textbf{B} \\ \hline 1.173 \pm 0.008 \\ \hline 1.224 \pm 0.023 \\ \hline 1.341 \pm 0.016 \\ \hline 1.308 \pm 0.116 \\ \hline 1.367 \pm 0.031 \end{array}$	$\begin{array}{c} \textbf{C} \\ \textbf{9} \\ \hline 1.311 \pm 0.008 \\ \hline 1.331 \pm 0.023 \\ \hline 1.360 \pm 0.017 \\ \hline 1.385 \pm 0.031 \\ \hline 1.346 \pm 0.027 \end{array}$	D           10           1.499±0.011           1.583±0.022           1.569±0.019           1.591±0.077           1.735±0.038	$\begin{array}{c} \textbf{R2} \\ \hline \textbf{11} \\ 0.076 \pm 0.008 \\ 0.077 \pm 0.017 \\ 0.110 \pm 0.011 \\ 0.035 \pm 0.011 \\ 0.034 \pm 0.013 \end{array}$
O4/06/08           Civil           Date           1           21/04/21           26/03/02           09/04/02           07/05/03           29/05/03           01/06/03	JD           2           2021           2360           2374           2767           2789           2792	Sbg           3           26           223           136           143           112	Nfr           4           5           2           4           4           4           4	0.84           FWHM           5           1.02           0.90           1.00           0.86           1.13           1.06	127           SNR           6           144           45           79           77           66           69	$\begin{array}{r} 1.203 \pm 0.003 \\ \hline V \text{ bane} \\ \hline A \\ \hline 7 \\ 1.000 \pm 0.007 \\ \hline 1.113 \pm 0.020 \\ \hline 1.108 \pm 0.012 \\ \hline 1.047 \pm 0.035 \\ \hline 1.043 \pm 0.023 \\ \hline 1.088 \pm 0.016 \end{array}$	$\begin{array}{r} 1.390 \pm 0.006 \\ \hline \textbf{B} \\ \hline \textbf{B} \\ \hline \textbf{B} \\ \hline 1.173 \pm 0.008 \\ \hline 1.224 \pm 0.023 \\ \hline 1.341 \pm 0.016 \\ \hline 1.308 \pm 0.116 \\ \hline 1.367 \pm 0.031 \\ \hline 1.294 \pm 0.020 \end{array}$	$\begin{array}{c} \textbf{C} \\ \textbf{9} \\ \hline 1.311 \pm 0.008 \\ \hline 1.331 \pm 0.023 \\ \hline 1.360 \pm 0.017 \\ \hline 1.385 \pm 0.031 \\ \hline 1.346 \pm 0.027 \\ \hline 1.370 \pm 0.020 \end{array}$	D           10           1.499±0.011           1.583±0.022           1.569±0.019           1.591±0.077           1.735±0.038           1.688±0.025	$\begin{array}{c} \textbf{R2} \\ \hline \textbf{R2} \\ \hline \textbf{0.076\pm0.008} \\ 0.077\pm0.017 \\ 0.110\pm0.011 \\ 0.035\pm0.011 \\ 0.034\pm0.013 \\ 0.050\pm0.012 \end{array}$
O4/06/08           Civil           Date           1           21/04/21           26/03/02           09/04/02           07/05/03           29/05/03           01/06/03           04/06/03	JD           2           2021           2360           2374           2767           2789           2792           2795	Sbg           3           26           223           136           143           112           117	Nfr           4           5           2           4           4           4           4           4           4           4           4	0.84           FWHM           5           1.02           0.90           1.00           0.86           1.13           1.06           1.04	127           SNR           6           144           45           79           77           66           69           73	$\begin{array}{r} 1.203 \pm 0.003 \\ \hline V \text{ bane} \\ \hline A \\ \hline 7 \\ 1.000 \pm 0.007 \\ \hline 1.113 \pm 0.020 \\ \hline 1.108 \pm 0.012 \\ \hline 1.047 \pm 0.035 \\ \hline 1.043 \pm 0.023 \\ \hline 1.088 \pm 0.016 \\ \hline 1.069 \pm 0.010 \end{array}$	$B$ $B$ $1.173\pm0.008$ $1.224\pm0.023$ $1.341\pm0.016$ $1.308\pm0.116$ $1.367\pm0.031$ $1.294\pm0.020$ $1.292\pm0.011$	c         9         1.311±0.008         1.331±0.023         1.360±0.017         1.385±0.031         1.346±0.027         1.370±0.020         1.404±0.012	D           10           1.499±0.011           1.583±0.022           1.569±0.019           1.591±0.077           1.735±0.038           1.688±0.025           1.653±0.016	-0.147 $\pm$ 0.002           R2           11           0.076 $\pm$ 0.008           0.077 $\pm$ 0.017           0.110 $\pm$ 0.011           0.035 $\pm$ 0.011           0.034 $\pm$ 0.013           0.050 $\pm$ 0.012           0.047 $\pm$ 0.011
O4/06/08           Civil           Date           1           21/04/21           26/03/02           09/04/02           07/05/03           29/05/03           01/06/03           04/06/03           17/06/03	JD           2           2021           2360           2374           2767           2789           2792           2795           2808	Sbg           3           26           223           23           136           143           112           117           164	N <sub>fr</sub> 4           5           2           2           4           4           4           4           4           4           4           4           4           4           4           4           4           4           4           4           4	5           1.02           0.90           1.00           0.86           1.13           1.06           1.04	127           SNR           6           144           45           79           77           66           69           73           52	$\begin{array}{r} 1.203 \pm 0.003 \\\hline V \text{ bane} \\\hline A \\\hline 1.000 \pm 0.007 \\\hline 1.113 \pm 0.020 \\\hline 1.108 \pm 0.012 \\\hline 1.047 \pm 0.035 \\\hline 1.043 \pm 0.023 \\\hline 1.088 \pm 0.016 \\\hline 1.069 \pm 0.010 \\\hline 1.093 \pm 0.013 \\\hline \end{array}$	$B$ $B$ $1.173\pm0.008$ $1.224\pm0.023$ $1.341\pm0.016$ $1.308\pm0.116$ $1.367\pm0.031$ $1.294\pm0.020$ $1.292\pm0.011$ $1.340\pm0.016$	$\begin{array}{c} \textbf{C} \\ \textbf{9} \\ \hline 1.311 \pm 0.008 \\ \hline 1.311 \pm 0.023 \\ \hline 1.360 \pm 0.017 \\ \hline 1.385 \pm 0.031 \\ \hline 1.346 \pm 0.027 \\ \hline 1.370 \pm 0.020 \\ \hline 1.404 \pm 0.012 \\ \hline 1.418 \pm 0.016 \end{array}$	D           10           1.499±0.011           1.583±0.022           1.569±0.019           1.591±0.077           1.735±0.038           1.688±0.025           1.653±0.016           1.639±0.020	-0.147 $\pm$ 0.002           R2           11           0.076 $\pm$ 0.008           0.077 $\pm$ 0.017           0.110 $\pm$ 0.011           0.035 $\pm$ 0.011           0.034 $\pm$ 0.013           0.050 $\pm$ 0.012           0.047 $\pm$ 0.011           0.035 $\pm$ 0.015
O4/06/08           Civil           Date           1           21/04/21           26/03/02           09/04/02           07/05/03           29/05/03           01/06/03           04/06/03           17/06/03           29/06/03	JD           2           2021           2360           2374           2767           2789           2792           2795           2808           2820	Sbg           3           26           223           23           136           143           112           117           164           153	Nfr           4           5           2           2           4	<b>FWHM 5</b> 1.02           0.90           1.00           0.86           1.13           1.06           1.04           1.02	127           SNR           6           144           45           79           77           66           69           73           52           71	$\begin{array}{r} 1.203 \pm 0.003 \\\hline V \text{ bane} \\ \hline \\ $	$B$ $B$ $1.173\pm0.008$ $1.224\pm0.023$ $1.224\pm0.023$ $1.341\pm0.016$ $1.308\pm0.116$ $1.367\pm0.031$ $1.294\pm0.020$ $1.292\pm0.011$ $1.340\pm0.016$ $1.368\pm0.018$	C         9         1.311±0.008         1.331±0.023         1.360±0.017         1.385±0.031         1.346±0.027         1.370±0.020         1.404±0.012         1.418±0.016         1.409±0.016	D           10           1.499±0.011           1.583±0.022           1.569±0.019           1.591±0.077           1.735±0.038           1.688±0.025           1.653±0.016           1.639±0.020           1.653±0.020	$\begin{array}{c} \textbf{R2} \\ \hline \textbf{R2} \\ \hline \textbf{11} \\ 0.076 \pm 0.008 \\ 0.077 \pm 0.017 \\ 0.110 \pm 0.011 \\ 0.035 \pm 0.011 \\ 0.034 \pm 0.013 \\ 0.050 \pm 0.012 \\ 0.047 \pm 0.011 \\ 0.035 \pm 0.015 \\ 0.044 \pm 0.011 \end{array}$
O4/06/08           Civil           Date           1           21/04/21           26/03/02           09/04/02           07/05/03           29/05/03           01/06/03           17/06/03           29/06/03           30/06/03	JD           2           2021           2360           2374           2767           2789           2792           2795           2808           2820           2821	Sbg           3           26           223           23           136           143           112           117           164           153           144	N <sub>fr</sub> 4           5           2           4           4           4           4           4           4           4           4           4           4           4           4           4           4           4           4	0.84           FWHM           5           1.02           0.90           1.00           0.86           1.13           1.06           1.04           1.02           0.89	SNR           6           144           45           79           77           66           69           73           52           71           66	$\begin{array}{r} 1.203 \pm 0.003 \\\hline V \text{ bane} \\ \hline \\ $	$B$ $B$ $1.173\pm0.008$ $1.224\pm0.023$ $1.341\pm0.016$ $1.308\pm0.116$ $1.367\pm0.031$ $1.294\pm0.020$ $1.292\pm0.011$ $1.340\pm0.016$ $1.368\pm0.018$ $1.363\pm0.010$	C         9         1.311±0.008         1.331±0.023         1.360±0.017         1.385±0.031         1.346±0.027         1.370±0.020         1.404±0.012         1.418±0.016         1.422±0.010	D           10           1.499±0.011           1.583±0.022           1.569±0.019           1.591±0.077           1.735±0.038           1.653±0.016           1.653±0.020           1.653±0.020           1.676±0.012	-0.147 $\pm$ 0.002           R2           11         0.076 $\pm$ 0.008         0.077 $\pm$ 0.017         0.110 $\pm$ 0.011         0.035 $\pm$ 0.011         0.035 $\pm$ 0.011         0.034 $\pm$ 0.013         0.050 $\pm$ 0.012         0.047 $\pm$ 0.011         0.035 $\pm$ 0.015         0.044 $\pm$ 0.011         0.029 $\pm$ 0.012
O4/06/08           Civil           Date           1           21/04/21           26/03/02           09/04/02           07/05/03           29/05/03           01/06/03           17/06/03           29/06/03           30/06/03           14/07/03	JD           2           2021           2360           2374           2767           2789           2792           2795           2808           2820           2821           2835	Sbg           3           26           223           136           143           112           117           164           153           144           765	N <sub>fr</sub> 4           5           2           4	0.84           FWHM           5           1.02           0.90           1.00           0.86           1.13           1.06           1.04           1.02           0.89           1.01	127           SNR           6           144           45           79           77           66           69           73           52           71           66           35	$\begin{array}{r} I.203 \pm 0.003 \\\hline V \text{ bane} \\ \hline \\ $	$\begin{array}{r} 1.390 \pm 0.006 \\ \hline \textbf{H} \\ \hline \textbf{B} \\ \hline \textbf{B} \\ \hline 1.173 \pm 0.008 \\ \hline 1.224 \pm 0.023 \\ \hline 1.341 \pm 0.016 \\ \hline 1.308 \pm 0.116 \\ \hline 1.367 \pm 0.031 \\ \hline 1.294 \pm 0.020 \\ \hline 1.292 \pm 0.011 \\ \hline 1.340 \pm 0.016 \\ \hline 1.368 \pm 0.018 \\ \hline 1.363 \pm 0.010 \\ \hline 1.342 \pm 0.039 \end{array}$	C         9         1.311±0.008         1.331±0.023         1.360±0.017         1.385±0.031         1.346±0.027         1.370±0.020         1.404±0.012         1.418±0.016         1.422±0.010         1.483±0.038	$\begin{array}{c} \textbf{D} \\ \hline \textbf{10} \\ \hline 1.499 \pm 0.011 \\ \hline 1.583 \pm 0.022 \\ \hline 1.569 \pm 0.019 \\ \hline 1.591 \pm 0.077 \\ \hline 1.735 \pm 0.038 \\ \hline 1.688 \pm 0.025 \\ \hline 1.653 \pm 0.016 \\ \hline 1.639 \pm 0.020 \\ \hline 1.653 \pm 0.020 \\ \hline 1.676 \pm 0.012 \\ \hline 1.630 \pm 0.043 \end{array}$	R2           11 $0.076\pm0.008$ $0.077\pm0.017$ $0.110\pm0.011$ $0.035\pm0.011$ $0.034\pm0.013$ $0.050\pm0.012$ $0.047\pm0.011$ $0.035\pm0.015$ $0.044\pm0.011$ $0.029\pm0.012$ $0.048\pm0.021$
O4/06/08           Civil           Date           1           21/04/21           26/03/02           09/04/02           07/05/03           29/05/03           01/06/03           04/06/03           17/06/03           29/06/03           30/06/03           14/07/03           17/07/03	JD           2           2021           2360           2374           2767           2789           2792           2795           2808           2820           2821           2835           2838	Sbg           3           26           223           136           143           112           117           164           153           144           765           252	N <sub>fr</sub> 4           5           2           4	<b>FWHM 5</b> 1.02           0.90           1.00           0.86           1.13           1.06           1.04           1.02           0.89           1.01	127           SNR           6           144           45           79           77           66           69           73           52           71           66           35           50	$\begin{array}{r} I.203 \pm 0.003 \\\hline V \text{ bane} \\ \hline \\ $	$\begin{array}{r} 1.390 \pm 0.006 \\ \hline \textbf{H} \\ \hline \textbf{B} \\ \hline \textbf{B} \\ \hline 1.173 \pm 0.008 \\ \hline 1.224 \pm 0.023 \\ \hline 1.341 \pm 0.016 \\ \hline 1.308 \pm 0.116 \\ \hline 1.367 \pm 0.031 \\ \hline 1.294 \pm 0.020 \\ \hline 1.292 \pm 0.011 \\ \hline 1.340 \pm 0.016 \\ \hline 1.368 \pm 0.018 \\ \hline 1.363 \pm 0.010 \\ \hline 1.342 \pm 0.039 \\ \hline 1.269 \pm 0.017 \end{array}$	C         9         1.311±0.008         1.331±0.023         1.360±0.017         1.385±0.031         1.346±0.027         1.370±0.020         1.404±0.012         1.418±0.016         1.422±0.010         1.483±0.038         1.410±0.018	$\begin{array}{c} \textbf{D} \\ \hline \textbf{D} \\ \hline \textbf{1.499 \pm 0.011} \\ 1.499 \pm 0.011 \\ 1.583 \pm 0.022 \\ 1.569 \pm 0.019 \\ 1.591 \pm 0.077 \\ 1.735 \pm 0.038 \\ 1.688 \pm 0.025 \\ 1.653 \pm 0.016 \\ 1.639 \pm 0.020 \\ 1.653 \pm 0.020 \\ 1.676 \pm 0.012 \\ 1.630 \pm 0.043 \\ 1.636 \pm 0.021 \end{array}$	-0.147 $\pm$ 0.002           R2           11         0.076 $\pm$ 0.008         0.077 $\pm$ 0.017           0.110 $\pm$ 0.011         0.035 $\pm$ 0.011         0.035 $\pm$ 0.013         0.050 $\pm$ 0.012           0.047 $\pm$ 0.011         0.035 $\pm$ 0.015         0.044 $\pm$ 0.011         0.029 $\pm$ 0.012           0.048 $\pm$ 0.021         0.048 $\pm$ 0.021         0.018 $\pm$ 0.016
O4/06/08           Civil           Date           1           21/04/21           26/03/02           09/04/02           07/05/03           29/05/03           01/06/03           04/06/03           17/06/03           29/06/03           30/06/03           14/07/03           19/07/03	JD           2           2021           2360           2374           2767           2789           2792           2795           2808           2820           2821           2835           2838           2840	Sbg           3           26           223           23           136           143           112           117           164           153           144           765           252           169	N         Fr           4         5           2         2           4         4           4         4           4         4           4         4           4         4           4         4           4         4           4         4           4         4           4         4           4         4           4         4           4         4           4         4	0.84           FWHM           5           1.02           0.90           1.00           0.86           1.13           1.06           1.04           1.02           0.89           1.01           1.08           1.13	127           SNR           6           144           45           79           77           66           69           73           52           71           66           35           50           61	$\begin{array}{r} I.203 \pm 0.003 \\\hline V \text{ bane} \\ \hline \\ P \\ I.000 \pm 0.007 \\\hline \\ I.113 \pm 0.020 \\\hline \\ I.108 \pm 0.012 \\\hline \\ I.047 \pm 0.035 \\\hline \\ I.043 \pm 0.023 \\\hline \\ I.043 \pm 0.023 \\\hline \\ I.043 \pm 0.023 \\\hline \\ I.093 \pm 0.016 \\\hline \\ I.093 \pm 0.013 \\\hline \\ I.102 \pm 0.014 \\\hline \\ I.119 \pm 0.008 \\\hline \\ I.035 \pm 0.028 \\\hline \\ I.196 \pm 0.016 \\\hline \\ I.153 \pm 0.016 \\\hline \end{array}$	$B$ $B$ $1.173\pm0.008$ $1.224\pm0.023$ $1.341\pm0.016$ $1.308\pm0.116$ $1.367\pm0.031$ $1.294\pm0.020$ $1.292\pm0.011$ $1.340\pm0.016$ $1.363\pm0.016$ $1.363\pm0.018$ $1.363\pm0.010$ $1.328\pm0.017$ $1.328\pm0.019$	c         9         1.311±0.008         1.331±0.023         1.360±0.017         1.385±0.031         1.346±0.027         1.370±0.020         1.404±0.012         1.418±0.016         1.422±0.010         1.483±0.038         1.410±0.018         1.402±0.017	D           10           1.499±0.011           1.583±0.022           1.569±0.019           1.591±0.077           1.735±0.038           1.688±0.025           1.653±0.016           1.639±0.020           1.653±0.020           1.676±0.012           1.630±0.043           1.636±0.021           1.663±0.023	-0.147 $\pm$ 0.002           R2           11         0.076 $\pm$ 0.008         0.077 $\pm$ 0.017         0.110 $\pm$ 0.011         0.035 $\pm$ 0.011         0.035 $\pm$ 0.012         0.047 $\pm$ 0.012         0.047 $\pm$ 0.011         0.035 $\pm$ 0.015         0.044 $\pm$ 0.011         0.029 $\pm$ 0.012         0.048 $\pm$ 0.021         0.018 $\pm$ 0.016         0.036 $\pm$ 0.014

27/02/04	3063	154	4	0.81	95	$1.150 \pm 0.005$	1.317±0.007	$1.415 \pm 0.007$	$1.536 \pm 0.007$	0.042±0.010
30/03/04	3095	345	4	1.00	67	$1.133 \pm 0.008$	1.312±0.010	$1.388 \pm 0.010$	1.616±0.012	0.036±0.012
10/04/04	3106	332	4	0.91	87	1.119±0.006	1.332±0.008	$1.433 \pm 0.007$	$1.584 \pm 0.009$	0.040±0.010
12/04/04	3108	152	2	1.03	91	$1.098 \pm 0.009$	1.366±0.011	1.427±0.012	$1.608 \pm 0.013$	0.041±0.008
13/04/04	3109	133	4	0.94	23	$1.124 \pm 0.006$	$1.331 \pm 0.007$	$1.423 \pm 0.007$	$1.583 \pm 0.008$	$0.043 \pm 0.007$
04/05/04	3130	137	3	1.08	98	$1.147 \pm 0.007$	$1.271 \pm 0.008$	$1.380{\pm}0.008$	$1.580 \pm 0.009$	0.051±0.008
09/05/04	3135	141	4	0.86	16	$1.145 \pm 0.005$	$1.292 \pm 0.005$	$1.378 \pm 0.005$	$1.601 \pm 0.006$	$0.043 \pm 0.007$
17/05/04	3143	133	3	1.01	80	$1.130\pm0.008$	$1.307 \pm 0.010$	$1.356 \pm 0.010$	$1.608 \pm 0.013$	$0.040 \pm 0.010$
23/05/04	3149	150	4	0.99	22	$1.096 \pm 0.007$	$1.243 \pm 0.008$	$1.362 \pm 0.009$	$1.598 \pm 0.010$	$0.039 \pm 0.007$
26/05/04	3152	374	4	0.88	67	$1.102 \pm 0.008$	$1.243 \pm 0.008$	$1.322 \pm 0.009$	$1.605 \pm 0.011$	0.037±0.011
08/06/04	3165	132	3	0.92	90	$1.063 \pm 0.005$	1.223±0.006	$1.357 {\pm} 0.007$	$1.547 \pm 0.008$	$0.034 \pm 0.008$
10/06/04	3167	118	3	1.25	97	$1.061 \pm 0.016$	1.233±0.020	$1.348 \pm 0.018$	1.521±0.024	$0.035 \pm 0.008$
11/06/04	3168	121	3	1.11	10	$1.056 \pm 0.007$	$1.229 \pm 0.008$	$1.355 \pm 0.008$	$1.528 \pm 0.010$	$0.033 \pm 0.008$
17/06/04	3174	120	3	1.07	96	$1.064 \pm 0.009$	1.241±0.012	$1.354 \pm 0.011$	$1.522 \pm 0.013$	$0.035 \pm 0.008$
19/06/04	3176	131	2	1.28	74	$1.047 \pm 0.022$	$1.215 \pm 0.026$	$1.355 \pm 0.024$	$1.532 \pm 0.032$	$0.032 \pm 0.010$
21/06/04	3178	130	4	1.19	98	$1.048 \pm 0.013$	$1.204 \pm 0.016$	$1.374 \pm 0.018$	$1.495 \pm 0.019$	$0.031 \pm 0.008$
23/06/04	3180	232	2	1.05	76	$1.083 \pm 0.014$	$1.192 \pm 0.016$	$1.384 \pm 0.016$	$1.522 \pm 0.019$	0.034±0.012
29/06/04	3186	2313	4	1.10	30	$1.061 \pm 0.025$	$1.200 \pm 0.030$	$1.372 \pm 0.031$	$1.494 \pm 0.035$	0.042±0.023
02/07/04	3189	1665	4	1.11	26	$1.052 \pm 0.023$	$1.187 \pm 0.027$	$1.325 \pm 0.028$	$1.423 \pm 0.029$	0.026±0.021
10/07/04	3197	222	4	1.06	72	$1.034 \pm 0.009$	1.238±0.011	$1.338 \pm 0.011$	1.399±0.012	0.045±0.011
04/08/04	3222	140	3	1.15	75	$1.019 \pm 0.014$	1.320±0.020	$1.337 \pm 0.017$	$1.418 \pm 0.017$	0.027±0.011
13/07/05	3565	500	4	1.34	58	1.115±0.029	1.158±0.032	$1.197 \pm 0.028$	$1.367 \pm 0.036$	0.137±0.015

- next, the individual sources of noise were added. The Clover Leaf model images were squeezed to the size of the original CCD frames, after being randomly centred at the sub-pixel scale. Then the sky background level, read-out noise and the signal noise were added. Finally, the resulting Clover Leaf images were again pixelated and interpolated in order to reproduce all known existing biases.

For each day of observation, we generated 500 simulated CCD frames and applied the developed procedure of adaptive PSF fitting to each of them. The calculated fluxes and consequently, the lensed component's magnitudes displayed a Gaussian distribution. For this reason, the photometric error bars associated with each observing night are defined as the standard deviations of the magnitudes out of the 500 corresponding simulations.

In our analysis, we ignored the presence of the gravitational lens near the centre of the system because it is too faint to be detected on our CCD frames [223].

The R-band and the V-band light curves of the lensed images of H1413+117 and the magnitude differences R1–R2 are presented in Figs 4.7 and 4.8 (see also Table 4.2). Note that the light curves of the lensed images are plotted relatively to the star R2. These curves clearly show that the fluxes of the H1413+117 lensed components are varying continuously and are correlated with the overall brightness change of the quasar (see Fig. 4.6). A detailed analysis of the light curves will be given after estimating the time delays, as well as a discussion of the colour variations.

## §4.5. Time delay calculation

The time delays ( $\Delta t$ ) constitute an important feature of gravitational lens systems [57,58]. These quantities are directly related to the gravitational potential of the lensing galaxy and to the geometrical properties of the Universe at large distances. A wide variety of algorithms have been developed for the determination of time delays in gravitational lens systems. The methods presented here are based on the chi-squared  $\chi^2$  [46,236] and dispersion  $D^2$  [72] minimizations. In general, all the techniques proceed in a similar way – they try to find how to match at best two given light curves. Usually, the chi-squared method is used in the case of wellsampled light curves with different types of interpolation between the inner points [197]. In the case of poorly sampled data, authors of [126] advise to use the dispersion methods such as the Pelt's algorithms [72]. These two techniques ( $\chi^2$ and  $D^2$ ) were also used in [225] to determine the time delay values between three pairs of the components of H1413+177 in 2008. We want to compare the results provided by the two approaches when applied to a moderately sampled data set like ours. The chi-squared  $\chi^2$  method that we used is well described in [46]. But in our case, we considered three values of the maximum gap ( $\delta = 20, 25$  and 30 d), within which linear interpolation was allowed. Furthermore, we have tested the dispersion method using two statistical values:  $D_2^2$  and  $D_3^2$ , which correspond to the formulas (7) and (8) of the paper in [72]. Here, we have used the same  $\delta = 20, 25$ and 30 d as the maximum distance between two observations which can be considered as sufficiently near.

We have considered the six possible pairs of light curves: AB, AC, AD, BC, BD and CD. As we can see on the plots (Figs 4.6 - 4.8), the brightness of the components varies with a high amplitude (up to 0.2 mag) over long time-scales. But the very long (several months) gaps between the seasons do not allow us to use the whole light curves. Because of unpredictable intrinsic and possible microlensing variations in the light curves of the components, we could not carry out any interpolation within these interseason gaps.

We thus decided to work with the individual fragments of the R-band light curves that correspond to the four observing seasons between 2003 and 2006. These seasons contain a sufficient number of data points (32 in 2004 and 20 in the others) to allow a relevant analysis. Next, we decided to apply a smoothing procedure to the light curves. We used a five-point median sliding filter, which corresponds, on average, to a 20-d filtering window. This allowed us to reduce the scattering in the photometry, i.e. to damp any sudden change in the light curves

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caused, first of all, by the conditions of observation and possible secondary fluctuations of the quasar-source, induced quasar micro-lensing effects, etc. (Fig. 4.9). This type of light curves is referred to as MFLC (median filtered light curves), contrarily to DLC (direct light curves), for which we did not use any filtering. We used both types of light curves separately for the calculation of the  $\Delta t$  values.



**Fig. 4.9.** Illustration of the five-point median filtering of the light curves during the 2003–2006 seasons. The background black points are the same as in Fig. 4.7.

To provide an estimation of the uncertainties and confidence level of the derived time delays, we adopted, as usually, a Monte Carlo method, i.e. we generated a large number of synthetic light curves and applied to each of them, the same calculation procedure to determine the corresponding time delay  $\Delta t$ . We used two different procedures to generate these artificial light curves:

(i) Our first procedure has consisted in creating synthetic light curves by simply adding a random number to the magnitude of each point of the real light curve. This number follows a normal distribution, centred on zero, with an FWHM equal to the photometric error bar of the point. This forms the group I of light curves.

(ii) We now describe the second procedure. Since we have previously calculated random magnitudes in accordance with a Gaussian distribution to estimate the photometric uncertainties, we have used these same data to re-create a new set of simulated light curves. The photometric error bars of each point of the light curves is the standard deviation of the set of the 500 random magnitudes corresponding to this point (see Section 5). Thus, we have generated another set of synthetic light curves by randomly choosing one of these calculated magnitudes and declared it as a point of the simulated light curve. To our opinion, this procedure of simulated light-curve generation is more reliable because it perfectly respects the magnitude distributions given by our adaptive PSF-fitting method: if the distribution is not perfectly normal, this fact will automatically be taken into account in the simulated light curves. This forms the group II of light curves.

We have compared the predictions following the analysis of these two sets of light curves. For each of these sets, we have generated 10 000 synthetic light curves. We have carried out  $\Delta t$  calculations covering a range of [-40, 40] d (and [-60, 20] d for the pairs BD and CD) with a step of 0.2 d, and values of magnitude differences between the components in the pairs covering appropriate ranges with a step of 0.002 mag.

As a result, we got a large number of independent and different (and contradicting, in many cases) values of time delays. These represent the average values of the main peaks in the  $\Delta t$  histograms. Note that we have obtained several hundred possible predictions of  $\Delta t$ : 4 years (2003–2006) × 6 pairs (AB, AC, AD, BC, BD and CD) × 3 formulas ( $\chi^2$ ,  $D_2^2$ ,  $D_3^2$ ) × 2 groups of simulated light curves (the set generated using normal distributions and the set generated using random

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choice) × 2 types of light curves (DLC and MFLC) × 3 values of  $\delta$ . Preliminary analysis of the time delay values showed that the observational season 2004 gives the most relevant results. See Table 4.3, where we have shown the only results of the group I (the group II gives almost the same results). In Fig. 4.9, we see that the A, B and C components varied in parallel in 2004, while D had a 'peculiar' behaviour probably induced by micro-lensing. However, we also found indirect evidence of micro-lensing in the component B on the basis of the colour index *V*–*R* (see Section above). But since this variation is not evident in the corresponding light curve, we have considered that the R-band light curve of the B component is clearly similar to those of A and C.

We now have thirty-six sets of solutions: according to the used formulas  $(\chi^2, D_2^2, D_3^2)$ , three values of  $\delta$ , two types of light curves (MFLC and DLC) and two groups of simulated light curves. To determine which of these time delay values are the most appropriate, we have adopted the main following criterion: a correct prediction must be self-consistent. We decided that a self-consistency test should be structured in two steps.

In the first step, we have to answer the question – are the  $\Delta t_{AB}$ ,  $\Delta t_{AC}$  and  $\Delta t_{BC}$  consistent with each other?, i.e.

$$\Delta t_{\rm BC} \cong \Delta t_{\rm AC} - \Delta t_{\rm AB}$$

In the second step of the self-consistency test, we look whether the  $\Delta t_{AB}$ ,  $\Delta t_{AC}$ ,  $\Delta t_{AD}$ ,  $\Delta t_{BC}$ ,  $\Delta t_{BD}$  and  $\Delta t_{CD}$  time delays are consistent with each other?, i.e.

$$\Delta t_{\rm BD} \cong \Delta t_{\rm AD} - \Delta t_{\rm AB}$$
$$\Delta t_{\rm CD} \cong \Delta t_{\rm BD} - \Delta t_{\rm BC}$$

In the first step, the answer 'yes' is mandatory. However, in the second step, we may get some 'anomaly' in  $\Delta t_{BD}$  and/or  $\Delta t_{CD}$  (difference between the estimated value and the expected one). These anomalies may be due to micro-lensing, and simply indicate that delays involving the light curve of D are not reliable (the formal errors do not incorporate a main systematic: micro-lensing).

**Table 4.3.** Estimates of the time delays (in days) obtained for the year 2004 as a function of the methods applied and the type of light curves analysed for three values of  $\delta = 20$ , 25 and 30 days. Only time delay values derived on the basis of light curves of the group\_I are shown here. DLC is direct simulated light curves, MFLC is median filtered light curves. If the histogram in the distribution of time delays does not allow to fix a peak, we mark it as Indefinite value (INDEF). 'no' or 'yes' are answers to the question of the first and second steps of the self-consistency test. In our  $\Delta t$  calculations, we assumed that:  $\Delta t_{XY} = \Delta t_Y - \Delta t_X$ .

			$\chi^2$			L	$D_{2}^{2}$			L	$D_{3}^{2}$	
Pair	MFL	С	DLC	2	MF	LC	DL	C	MFI	LC	DL	C
						$\delta = 20 \text{ day}$	/S					
$\Delta t_{\mathrm{AB}}$	$-17.8\pm2.2$		21.4±4.5		$-15.8 \pm 4.3$	-	$-11.1\pm1.7$		$-15.6\pm4.3$		$-11.2\pm1.7$	
$\Delta t_{\rm AC}$	$-19.7 \pm 4.4$		-15.1±2.3		$-19.6\pm2.1$		$-11.3\pm0.8$		$-19.4{\pm}2.1$		$-11.3 \pm 0.8$	
$\Delta t_{\rm AD}$	$-27.4\pm0.7$		-27.1±0.7		$30.3 \pm 2.9$		30.3±2.4		$30.6 \pm 2.9$		$30.7 \pm 2.4$	
$\Delta t_{\rm BC}$	$-16.2\pm6.1$	No	$-0.40\pm1.3$	No	$-24.5\pm4.5$	No	INDEF	No	$-28.5 \pm 4.5$	No	INDEF	No
$\Delta t_{\rm BD}$	$-11.3\pm0.5$	Yes	$-14.1\pm0.2$	No	$38.3 \pm 2.5$	Perhaps	34.6±1.5	Perhaps	38.1±2.5	Perhaps	34.7±1.5	Perhaps
$\Delta t_{\rm CD}$	$-13.4{\pm}1.1$	No	$-13.4\pm0.6$	Yes	$39.4{\pm}1.8$	No	$-12.9\pm0.4$	No	$38.8 \pm 1.8$	No	$-12.9{\pm}0.4$	No
02					(	$\delta = 25  \text{day}$	/S					
$\Delta t_{AB}$	-17.7±2.4		21.4±4.6		$-14.3\pm7.1$	-	-11.1±1.7		$-14.2\pm7.1$		$-11.1\pm1.7$	
$\Delta t_{\rm AC}$	-21.7±3.6		-15.1±2.2		-19.7±1.9		-11.3±0.9		$-19.7 \pm 1.9$		$-11.3\pm0.9$	
$\Delta t_{\rm AD}$	$-27.3\pm0.7$		-26.9±1.1		$30.6 \pm 2.6$		30.3±2.4		31.3±2.2		31.1±2.1	
$\Delta t_{\rm BC}$	$-18.3\pm3.1$	No	$-0.3{\pm}1.6$	No	$-24.0\pm4.2$	No	INDEF	No	$-24.1\pm4.1$	No	INDEF	No
$\Delta t_{\rm BD}$	$-27.3\pm0.4$	No	$-27.8\pm0.2$	No	$-0.7{\pm}1.8$	No	$33.9 \pm 0.8$	Perhaps	$2.1 \pm 1.6$	No	$34.0\pm0.5$	Perhaps
$\Delta t_{\rm CD}$	$-27.8\pm0.2$	No	$-27.4\pm0.3$	Yes	33.3±1.8	Perhaps	$-12.9\pm0.4$	No	$-27.2\pm0.9$	No	$-12.9\pm0.4$	No
CD CD						$\delta = 30  \mathrm{day}$	/S					
$\Delta t_{AB}$	$-17.4\pm2.1$		$-14.0\pm5.1$		$-16.8\pm5.1$	-	-11.1±1.5		$-16.8\pm5.1$		-11.1±1.5	
$\Delta t_{\rm AC}$	$-18.9\pm2.8$		$-15.0\pm2.4$		$-19.6\pm2.3$		$-11.3\pm0.8$		$-19.6\pm2.3$		$-11.3\pm0.8$	
$\Delta t_{\rm AD}$	$28.8 \pm 0.7$		$29.2 \pm 0.8$		$30.4 \pm 2.9$		30.3±2.5		$30.4 \pm 2.9$		$30.3 \pm 2.5$	
$\Delta t_{\rm BC}$	$-1.0\pm2.9$	Yes	$-0.9\pm0.3$	Yes	$-24.2\pm4.3$	No	$-28.3\pm6.4$	No	$-24.2\pm4.3$	No	$-28.3\pm6.4$	No
$\Delta t_{\rm BD}$	28.5±0.6	No	$-2.1\pm0.9$	No	$0.0{\pm}3.6$	No	33.9±0.7	Perhaps	$0.0{\pm}3.6$	No	33.9±0.7	Perhaps
$\Delta t_{\rm CD}$	28.9±1.5	Yes	$-14.0\pm0.5$	No	INDEF	No	INDEF	No	INDEF	No	INDEF	No

A comparison between the measured and expected time delays for the BC, BD and CD pairs is shown in Table 4.3 (see the answers 'yes' or 'no'). These results suggest that the  $\chi^2/MFLC/\delta = 30/\text{group}_I$  approach works much better than the other ones. Therefore, we have adopted the latter set as the final time delays (see Table 4.4). However, it should again be noted that the uncertainty in the  $\Delta t_{AD}$ ,  $\Delta t_{BD}$  and  $\Delta t_{CD}$  time delays is likely underestimated, because when calculating those time delays we did not take into account the effect of micro-lensing.

The change in the flux of the light curves that occurred at about 3150JD allowed us to derive self-consistent values for the time delays. During these days, the brightness of the components increased by ~0.06 mag during about 1 month with a rate of  $1.5 \times 10^{-3}$  mag day<sup>-1</sup> (see Fig. 4.9), while the average rate in brightness variation for the individual season is about  $6 \times 10^{-4}$  mag day<sup>-1</sup>. Thus, the predictions from the MFLC light curves (especially for the 2004 season) are more relevant than those of the direct light curves. Time delay and appropriate magnitude difference results are summarized in Table 4.4. According to the results of this table, we conclude that the proposed method for generating synthetic light curves (group\_II) works as well as the classic method and leads to quite similar results.

**Table 4.4.** Final estimates of the time delays  $\Delta t$  (in days) and magnitude differences  $\Delta m$  (in mag) on the basis of the  $\chi^2/MFLC/\delta = 30$  days / approach with estimates of the measurement accuracy. On the left, results for the group\_I synthetic light curves are given. On the right, the same but for group\_II. Here the B and C images are leading, then comes A, and at last, the D component is trailing the most.

	Group I of	light curves	Group II of light curves			
Pair	$\Delta t$	$\Delta m$	$\Delta t$	$\Delta m$		
$\Delta t_{\mathrm{AB}}$	$-17.4\pm2.1$	$0.148 \pm 0.003$	$-18.2\pm2.0$	$0.149 \pm 0.004$		
$\Delta t_{\rm AC}$	$-18.9 \pm 2.8$	$0.309 \pm 0.004$	$-18.1\pm2.5$	$0.309 \pm 0.004$		
$\Delta t_{ m AD}$	$28.8 \pm 0.7$	$0.414 \pm 0.003$	$28.6 \pm 0.6$	$0.415 \pm 0.003$		
$\Delta t_{ m BC}$	$-1.0{\pm}2.9$	$0.159 \pm 0.004$	$-0.5\pm2.7$	$0.159 \pm 0.004$		
$\Delta t_{ m BD}$	$28.5 \pm 0.6$	$0.277 \pm 0.003$	$-0.4{\pm}1.3$	$0.279 \pm 0.004$		
$\Delta t_{\mathrm{CD}}$	28.9±1.5	$0.115 \pm 0.003$	$28.6 \pm 0.8$	$0.115 \pm 0.004$		



**Fig. 4.10.** Combined light curve of H1413+117 for the MFLC case. These plots are made from the B, C, D (top) and B, C (bottom) light curves shifted in time and magnitude with respect to the A component. The time delay values are those in Table 4.4. But magnitude shifts between different seasons are different, probably due to micro-lensing effects (see text below).

In Fig. 4.10, we have illustrated the combined light curve of H1413+117 corrected for the predicted time delays (Table 4.4, left-hand side) and magnitude shifts (the time delays obtained by the formula  $\chi^2$  and  $\delta = 30$  d on the basis of the group\_I light curves). Because of active variability of the components, the magnitude differences between the components are not constant, and vary from

season to season. For example, in 2003  $\Delta m_{AB} = 0.198$ ,  $\Delta m_{AC} = 0.364$ ,  $\Delta m_{AD} = 0.557$ ; in 2004  $\Delta m_{AB} = 0.148$ ,  $\Delta m_{AC} = 0.309$ ,  $\Delta m_{AD} = 0.414$ ; in 2005  $\Delta m_{AB} = 0.103$ ,  $\Delta m_{AC} = 0.328$ ,  $\Delta m_{AD} = 0.370$ ; in 2006  $\Delta m_{AB} = 0.110$ ,  $\Delta m_{AC} = 0.318$ ,  $\Delta m_{AD} = 0.516$ . These magnitude values correspond to the peaks in the magnitude shift distributions.

As can be seen from Tables 4.3 and 4.4, the method of generating synthetic light curves based on randomly simulated magnitudes has a right to exist, because this approach leads to quite comparable results as those derived by the classical method based on the normal distribution with a width equal to the photometric error bars.

# §4.6. Microlensing and colour variations

The long-term light curves indicate the presence of micro-lensing effects, sometimes displaying a large amplitude with respect to intrinsic variability. In Figs 4.7–4.9, we see that the maximum amplitude of the component variations is also of the order ~0.15 mag (see the A component between 2002 and the beginning of the 2003 season). The brightness variability of the components is undoubtedly due to a superposition of some intrinsic variability of the quasar and non-correlated micro-lensing effects. Particularly noticeable changes are visible for the D component, during the seasons 2004 and 2005: its brightness increased by 0.1 mag.

At the bottom of Fig. 4.11, the combined light curves for 2004 and their fifth order polynomial fits are presented. We fitted the A + B + C and D curves separately. The difference between those light curves is interpreted as the signature of a micro-lensing effect for the D component. In general, gravitational lens systems are affected by micro-lensing with induced variability at the level of  $10^{-4}$  mag  $\cdot$  day<sup>-1</sup> [4,46,154,225]. But here we found out one of the fastest micro-lensing effects with a rate of  $10^{-3}$  mag  $\cdot$  day<sup>-1</sup>. As a consequence, the D component magnitude became almost comparable to that of the C component in
2005. In 2007, apparently, the induced micro-lensing flux increase stopped and the magnitude of the D component came back to its 'quasi-original' state.



Fig. 4.11. Combined light curves of H1413+117 during the 2004 season with appropriate  $\Delta t$  and  $\Delta m$  shifts and smoothed curves fitted with fifth order polynomials.

The C component shows smaller light variations in 2005, while all the other components increased in brightness. It seems that its amplification decreased during this period. Furthermore, the magnitude change between 2005 and 2006 (as well as between 2006 and 2007) is smaller, contrary to the other components (except D for the latter). Perhaps another amplifying micro-lensing event occurred at this time, during 2006 or 2007, or it might be the same event, spreading over 3 years.

The flux slope of the B component in 2003 is smaller than those of the other components. Perhaps it is due to a micro-lensing deamplification event that occurred before 2003 and ended sometime after our observations began, or a short micro-lensing amplification event occurring at the end of the observations for this year.



**Fig. 4.12.** *V*–*R* colour curves of the components. The colour values are near -0.1 mag. For clarity, the B, C and D colour curves have been shifted down by 0.1, 0.3 and 0.4 mag, respectively.

Another evidence of micro-lensing can be brought up after inspection of the colour light curves of the lensed components. Unusually brightness variations due to micro-lensing are chromatic, such as the intrinsic variations of the quasar. Therefore, colour curves have to show, after correction for the time delays, almost identical variations for each lensed component, except during micro-lensing events. And as we can see in Fig. 4.12, the colours of the B and D components were changing during 2004, becoming bluer, but not those of the other components. Brightness variation of the B component is larger in the V band than in the R band, which is also confirmed by the plot of the colour light curves (see Fig. 4.12). Their colour changes are unrelated because the slope of the B colour curve is clearly different from the slope shown by the D colour one. Furthermore, these colour curves confirm that the B component displayed a micro-lensing event

as strong as the D-component event, although it was less visible in the light curves. Therefore, we confirm the presence of chromatic micro-lensing events in the Clover Leaf.

The A component seems to be the most stable. Its light and colour behaviours during each season are similar to the behaviour of at least one of the other components. The influence of micro-lensing in the A component seems to be the least.

In principle one can calculate the masses of the lenses responsible for the microlensing from the total time scales of brightness variations and the transverse velocities of the compact objects. This approach was used by Pelt et al. [279] and later by Paraficz et al. in [111] to estimate the microlens masses in SBS1520+530, FBQ 0951+2635 and Q0957+561. It can be assumed a microlens is some compact object belonging to the population of the lensing galaxy and the radiation of the D component (as a source) crosses the Einstein ring of this microlens. Then, knowing the distance to the lens, we can estimate the lower limit for mass of the microlens.

The mass of the microlens is related to the angular size of the Einstein ring:

$$\theta = \sqrt{\frac{4GM}{c^2} \frac{D_{LS}}{D_L D_S}}$$

where G – gravitational constant, c – speed of light,  $D_S$ ,  $D_L$ , and  $D_{LS}$  – distance of source, microlens and between source and microlens, respectively, M – microlens mass. Since microlens is part of the population of the galaxy-lens and in general orbits around the center, it has a certain speed v, corresponding to the dispersion of velocities in the galaxy, the value of which is also not known. But it is possible to use the dispersion value ~200 – 600 km/s – typical for stellar population in galaxies. The linear size of the Einstein ring can be expressed as:

$$R = \theta \cdot D_L = v \cdot t \tag{4.12}$$

where t – the duration of the microlensing event. In the case of H1413+117, the duration of the microlensing event in 2004-2006 years about of 2.5 years was found from the comparative analysis of the brightness curves of components A and D. Substituting the expression for the angular radius of the Einstein ring in (4.12), we find an approximate formula for the mass of the microlens:

$$M = \frac{v^2 t^2 c^2}{4G} \frac{D_s}{D_{LS} D_L}$$
(4.13)

As we noted above, based on the model construction, we found the value of the red shift for the lensing galaxy. A number of formulas and methods can be used to calculate distances at cosmological distances, see e.g. [25,26]. In addition, it should be borne in mind that these calculations dependent on the value of the Hubble constant. We used the result of the Planck Collaboration project for H<sub>0</sub> [237], according to which the Hubble constant is  $H_0 = 67.8 \pm 0.9 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ .

v	$M_{\odot}$	$M_{Jup}$	ν	$M_{\odot}$	$M_{Jup}$
200	0.0019	2.0047	250	0.0030	3.1324
300	0.0043	4.5106	350	0.0059	6.1394
400	0.0077	8.0188	450	0.0097	10.1489
500	0.0120	12.5295	550	0.0145	15.1606
600	0.0172	18.0424	650	0.0202	21.1748

Table 4.5. Estimation of the microlens mass for different velocities in H1413+117

As we can see from the table 4.5, the estimated mass of the microlens stricly depends on the dispersion of velocities in the lensing galaxy. It turns out that the minimum possible mass of the microlens in the H1413+117 is equal to  $\sim 2 \cdot M_{Jup}$ . If we take the average dispersion of velocities for distant galaxies equal to about 450 km/s, the possible mass of the microlens will be equal to 0.01 solar mass or 10 masses of Jupiter. The same calculations for possible mass of microlenses in double quasars SBS1520+ and FBQ0951+2635 where microlensing

lasts for several decades, shown the microlensing in these cases is a body with stellar mass  $(0.2 \div 0.4) \cdot M_{\odot}$ .

### §4.7. Lens model and lens redshift

The theoretical time delay between a pair of lensed images depends on the deflector mass profile through the adopted lens model parameters but also on the cosmological scalefactor. The latter is a function of the source redshift  $z_s$ , the lens redshift  $z_l$  and the cosmology (essentially the Hubble parameter  $H_0$  and to a lesser extent on  $\Omega_m$  for the case of a flat Universe, see e.g. Schneider, Ehlers & Falco 1992). Refsdal was the first to demonstrate that  $H_0$  can be deduced from the expression of time delays between pairs of lensed images, as long as  $z_s$ ,  $z_l$  and the lens model are known [57,58]. For the case of H1413+117, the main lens is a very faint object ( $F_{160W} > 20.52 \text{ mag}$ ,  $F_{180M} > 22.18 \text{ mag}$ , [223]) for which the redshift lacks a final and reliable estimation so far. However, the Planck Collaboration has recently proposed a robust estimation for  $H_0$  and  $\Omega_m$  based on full-mission Planck observations of temperature and polarization anisotropies of the cosmic microwave background radiation [237]. Therefore, adopting a lens model, the value  $z_s = 2.558$  (e.g. in [238]) for the redshift of the source, as well as a flat universe characterized by  $\Omega_{\rm m} = 0.308 \pm 0.012$  and  $H_0 = 67.8 \pm$  $0.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , the expression of the theoretical time delays only depends on  $Z_l$ .

We propose then to update the value of  $z_l$  by determining the optimal set of parameters which fits the available observable quantities, that is to say the four lensed image positions, the three flux ratios and the three time delays. Such an approach has already been carried out in [225] for H1413+117 and they obtained  $z_l = 1.88^{+0.09}_{-0.11}$ . We followed the same procedure but we used a more recent value for  $H_0$  and the  $\chi^2/MFLC/\delta = 30/group_I$  set of time delays derived in the present work (see Table 4.4). The lensed image positions come from HST wide field planetary camera (WFPC)/WFPC2 images [213] and are reported in Table 4.6. We

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have adopted the mid-IR flux ratios given in [224] and reported in the third column of Table 4.6. Although the image fluxes are likely affected by micro-lensing (see e.g. [7,216,239] and extinction [213], they claim that the 11  $\mu$ m observations which led to the mid-IR flux ratios should be very little affected by both these effects.

The lens scenario consists of a point-like source that is lensed by a combination of three components:

(i) the main lensing galaxy G1 modelled as a singular isothermal ellipsoid (SIE) characterized by its relative position  $(\Delta \alpha_{G1}, \Delta \delta_{G1})$  with respect to the lensed image A, mass scale  $b_{G1}$ , ellipticity  $e_{G1}$  and position angle  $\theta_{eG1}$ ,

(ii) the lensing galaxy G2 (object No. 14 in [222]) modelled as a singular isothermal sphere (SIS) characterized by its relative position ( $\Delta \alpha_{G2}, \Delta \delta_{G2}$ ) with respect to the lensed image A and mass scale  $b_{G2}$ ,

(iii) an external shear to account for the presence of galaxy overdensities and other contributions, characterized by its strength  $\gamma_{\text{ext}}$  and orientation  $\theta_{\gamma \text{ ext}}$ .

**Table 4.6.** The lensed image (A–D) and deflector positions along with the mid-IR flux ratios for the H1413+117 system. The values of the lensed image positions come from [213], while the deflector position comes from [224].

Component	ΔRA	$\Delta \text{DEC}$	Mid-IR
-	(in arcsec)	(in arcsec)	
А	0.0	0.0	≡1.00
В	$0.744 \pm 0.003$	$0.168 \pm 0.003$	$0.84{\pm}0.07$
С	$-0.492{\pm}0.003$	$0.713 {\pm} 0.003$	$0.72{\pm}0.07$
D	$0.354 \pm 0.003$	$1.040 \pm 0.003$	$0.04{\pm}0.06$
G1	$-0.142 \pm 0.020$	$0.561 \pm 0.020$	
G2	-1.870	4.140	

Since the position of the gravity centre of the main lens is not precisely known, it has been considered as an additional parameter which is allowed to vary during the fit. The adopted G2 galaxy position with respect to the lensed image A is  $(\Delta \alpha, \Delta \delta) = (-1.87, 4.14)$  arcsec [221]. Let us note that since we consider G1 and G2 to be lying in the same plane, the lens redshift  $z_l$  refers to the pair G1–G2. Using the GRAVLENS software package [240], the best solutions for the  $\chi^2/MFLC/\delta = 30/\text{group}$  set of time delays (see Table 4.3) is  $z_l = 1.95^{+0.06}_{-0.10}$  with  $\chi^2_{\text{tot}} = 3.20$  for 3 degrees of freedom (dof). The model has 14 constraints (the four lensed image positions, the three flux ratios and the three time delays) and 11 parameters (see Table 4.7, plus the point-like source position). Contrary to results of [224] and [225], we do not assume any priors on the ellipticity and position of G1, and the strength of the external shear.

Doromotor	[225]	Our work	Our work	
Farameter		$\chi^2/MFLC/\delta=30/group_I$	idem + relaxing $\sigma$ ( $\Delta t_{AD}$ )	
$\chi^2$	7.5/7	3.20 / 3	2.61 / 3	
$\chi^2_{\rm image}$	_	0.01	0.01	
$\chi^2_{\rm flux}$	_	0.94	0.71	
$\chi^2_{\rm delay}$	—	0.82	0.46	
$\chi^2_{G1}$	_	1.42	1.43	
$b_{G1}$	0.68	0.56	0.56	
$\Delta \alpha_{G1}$	-0.165	- 0.163	-0.164	
$\Delta \delta_{ m G1}$	0.552	0.550	0.552	
$e_{G1}$	0.28	0.32	0.31	
$\theta \ e_{G1}$	- 37.6	- 39.7	- 39.2	
$b_{G2}$	0.45	0.38	0.47	
$\Delta \alpha_{G2}$	$\equiv -1.87$	$\equiv -1.87$	$\equiv -1.87$	
$\Delta \delta_{G2}$	$\equiv 4.14$	≡ 4.14	≡ 4.14	
$\gamma_{\rm ext}$	0.11	0.13	0.12	
$\theta \gamma_{\rm ext}$	45.4	44.7	46.1	
$Z_{l}$	1.88	1.95	1.97	

**Table 4.7.** Modelling results and comparison with previous work. The parameters  $b_{\text{Gk}}$ ,  $\Delta \alpha_{\text{Gk}}$  and  $\Delta \delta_{\text{Gk}}$  are given in arcsec, and,  $\theta_{e \text{ G1}}$  and  $\theta_{\gamma \text{ ext}}$  in degree.

In Section 4.6, we found evidence of significant micro-lensing variability in the light curve of the component D. Such an effect could bias our determination of  $\Delta t_{AD}$ . To quantify this impact on the determination of  $z_l$ , we relaxed the time delay constraint between A and D, i.e. we did not take into account  $\Delta t_{AD}$ , and obtained  $z_l = 1.97^{+0.07}_{-0.11}$  with  $\chi^2_{tot} = 2.61$  for dof = 3.

In Fig. 4.13, we have illustrated the total  $\chi^2_{tot}$  as a function of  $z_l$  (thick solid line) both for the  $\chi^2/MFLC/\delta = 30/\text{group}_I$  set of time delays with (right-hand panel) and without (left-hand panel) relaxing the time delay constraint on *D*. The total  $\chi^2_{tot}$  has been broken down into four contributions, respectively for the lensed image positions ( $\chi^2_{image}$ , dashed lines), the flux ratios ( $\chi^2_{flux}$ , dash-dotted lines), the time delays ( $\chi^2_{delay}$ , thin solid lines) and the G1 position ( $\chi^2_{G1}$ , dotted lines), see the first part of Table 4.8.

**Table 4.8.** Modelled image and G1 positions, flux ratios and time delays derived from the best solution respectively for the  $\chi^2/MFLC/\delta = 30/\text{group}_I$  set of time delays, with or without relaxing the time delay constraint on *D*.

Parameter	Our work $\chi^2/MFLC/\delta=30/group_I$	Our work idem + relaxing $\sigma (\Delta t_{AD})$
Image positions (in arcs	ec)	
А	[0.0, 0.0]	[0.0, 0.0]
В	[0.744, 0.168]	[0.744, 0.168]
С	[-0.492, 0.713]	[-0.492, 0.713]
D	[0.354, 1.040]	[0.354, 1.040]
Image positions (in arcs	ec)	
G1	[-0.163, 0.550]	[-0.164, 0.552]
Flux ratios		
А	$\equiv 1.0$	$\equiv 1.0$
В	0.81	0.79
С	0.74	0.74
D	0.45	0.42
Time delay (in days)		
$\Delta t_{AB}$	-15.6	-16.4
$\Delta t_{\rm AC}$	-19.6	-20.3
$\Delta t_{\rm AD}$	28.9	32.7

The  $1\sigma$  (68 per cent) and  $2\sigma$  (95 per cent) confidence intervals are represented respectively by the dark and whole shaded areas. In both cases, the fit is dominated in the  $1\sigma$  confidence interval by the main galaxy G1 position. The contribution of  $\chi^2_{\text{flux}}$  is also meaningful and clearly shapes the total  $\chi^2_{\text{tot}}$  curve. Since the fit allows to reproduce almost perfectly the lensed image positions, the  $\chi^2_{\text{image}}$  contribution to the total  $\chi^2$  is almost negligible regardless of  $z_l$ . We have tested the goodness of the fit for lens redshift values down to 1. Although the total  $\chi^2_{tot}$  was not satisfactory, we noted that the lensed image positions were equally well reproduced. This tends to confirm that the lensed image positions can be easily reproduced with a large range of model parameters but also with different families of mass profiles. In particular, authors of [241,242] have theoretically pointed out the existence of a general transformation of the source plane, the socalled source-position transformation (SPT) whose well-known mass sheet degeneracy (MSD, [243]) is a special case. The SPT defines a new family of deflection laws and leaves almost invariant the lensed image positions and their flux ratios. Since no complete quantitative analysis describing how the SPT affects the behaviour of the time delays exists yet, it is difficult to infer how it would impact on the value of the lens redshift. However, accounting for a uniform external convergence (MSD) characterized by  $k_{\text{ext}} \sim 0.1$  kext ~ 0.1, authors of [225] have estimated that the value of the lens redshift should increase by 3 per cent at most.

The lens redshift estimations obtained for the  $\chi^2/MFLC/\delta = 30/\text{group}\_I$  set of time delays, with and without relaxing the time delay constraint on D, are consistent. The improvement in  $\chi^2_{\text{tot}}$  when relaxing the time delay constraint on D comes respectively from the time delays ( $\Delta\chi^2_{\text{delay}} = 0.36$ ) and the flux ratios ( $\Delta\chi^2_{\text{flux}} = 0.23$ ), see Table 4.7. Furthermore, the lens redshift estimations are also in good agreement with the value derived in [225],  $z_l = 1.88^{+0.09}_{-0.11}$ , even though our results constitute a slight improvement in terms of reduced total  $\chi^2_{\text{tot}}$ .

In addition, several arguments proposed in [225] remain valid, that is to say that  $z_l$  is in good or reasonable agreement with: (1) the G2 photometric redshift of about 2 derived in [221]; (2) the most distant detected overdensity at  $z_{ove} =$  $1.75 \pm 0.2$  [221,244]; (3) and absorbers at  $z_{abs} \sim 1.87$  [212]. In addition,  $z_l \sim 1.95$ is in agreement with the fact that the main galaxy G1 has been detected in HST/NICMOS-2 images with the near-IR H-band F<sub>160W</sub> (band-width = [1.4, 1.8]  $\mu$ m) and F<sub>180M</sub> (bandwidth = [1.76, 1.83]  $\mu$ m) filters [223] but not on HST/WFPC2 images, neither with the near-IR I-band F<sub>814W</sub> (bandwidth = [0.7, 1]  $\mu$ m) nor the near-IR R-band F<sub>702W</sub> (bandwidth = [0.60.8]  $\mu$ m) filters [213].



**Fig. 4.13.** Estimation of the lens redshift  $z_l$  for the  $\chi^2/MFLC/\delta = 30/\text{group}\_I$  set of time delays, without (left-hand panel) and with (right-hand panel) relaxing the time delay constraint on D. The total  $\chi^2_{tot}$  (thick solid lines) has been broken down into four contributions, respectively  $\chi^2_{image}$  (dashed lines),  $\chi^2_{flux}$  (dash-dotted lines),  $\chi^2_{delay}$  (thin solid lines) and  $\chi^2_{G1}$  (dotted lines). We have also represented the  $1\sigma$  (dark shaded area) and  $2\sigma$  (whole shaded area) confidence intervals.

Indeed, for  $z_l = 1.95$ , the galaxy spectrum Ca<sub>II</sub> break occurs at  $\lambda_{break} \sim 1.18 \,\mu\text{m}$  while, for  $z_l = 1$ , it occurs at  $\lambda_{break} \sim 0.8 \,\mu\text{m}$  [245]. Since the main galaxy G1 should be brighter at wavelengths  $\lambda > \lambda_{break}$ , it is clear that, for

the case of  $z_l \sim 1.95$ , its detection would likely be easier with the F<sub>160W</sub> and F<sub>180M</sub> filters, both with  $\lambda > 1.18 \,\mu\text{m}$ , rather than with the F<sub>814W</sub> and F<sub>702W</sub> filters, both with  $\lambda < 1.18 \,\mu\text{m}$ . Conversely, a main galaxy G1 at  $z_l = 1$  would be already brighter at wavelengths  $\lambda > 0.8 \,\mu\text{m}$ , and would have likely been detected with the F<sub>814W</sub> filter, which is actually not the case.



**Fig. 4.14.** The total  $\chi^2_{tot}$  map illustrating the degeneracy between two model parameters: the ellipticity of the main lens galaxy G1 and the strength of the external shear.

For both the  $\chi^2/MFLC/\delta = 30$ /group\_I set of time delays with and without relaxing the time delay constraint on D, the corresponding modelling results for the best solution are reported in the second part of Table 4.7. The corresponding results for the modelled image positions, flux ratios and time delays are reported in Table 4.8. The relative position and position angle of G1 as well as the shear orientation are in very good agreement with the ones obtained by [225]. As mentioned in the latter paper, a weak lensing analysis performed by [244] indicated a shear direction of 45° and a shear strength smaller than 0.17, which is in very good agreement with our modelling results, respectively ( $\gamma_{ext}$ ,  $\theta_{ext}$ ) = (0.13, 44.7°) for the  $\chi^2/MFLC/\delta = 30$ /group\_I set of time delays and ( $\gamma_{ext}$ ,  $\theta_{ext}$ ) = (0.12, 46.1°) when relaxing the time delay constraint on D. However, we note that while the G1 mass scale is significantly shorter, the external shear and the ellipticity are slightly larger than the ones given in [225]. Since the net quadrupole

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moment can have contributions from both the main lens galaxy ellipticity and the presence of an external shear, there might be a substantial degeneracy between these two parameters [147]. To point out this degeneracy, we have illustrated in Fig. 14 the total  $\chi^2_{tot}$  map for ranges  $e_{G1} \in [0.24, 0.37]$  and  $\gamma_{ext} = \in [0.06, 0.18]$  which corresponds to the solution  $z_l = 1.97$ . It is clear that our solution is fully compatible with the one given in [225]. Same results were obtained for our solution  $z_l = 1.95$ .

### §4.8. Summary and discussion on Chapter IV

*Observations of H1413+117.* In this article, we have presented for the first time an analysis of long-term photometric light curves derived for the lensed images of the quadruply imaged quasar H1413+117 (Clover Leaf). This object was observed in the V and R spectral bands with the 1.5-m telescope of the Maidanak observatory from 2001 to 2008.

Due to the compactness of this GLS, it was very challenging to retrieve the photometric flux of its four individual lensed components. On the other hand, there is no need for taking into account the light contamination from the lensing galaxy, as its contribution to the total flux as a whole and for the individual components in the Maidanak CCD frames is negligibly small (of the order of 22 mag, [223]).

A very efficient photometric method. To derive the magnitudes of the lensed QSO images, we have developed a new photometric technique: the adaptive PSF-fitting technique. First, this method only works with a numerical PSF and does not require an analytical representation. Secondly, there is no need for the assumption of the PSF invariance over large regions of the CCD frame. To calculate the fluxes of the lensed components, we assumed the PSF invariance over only the GLS (i.e. less than 2 arcsec). The problem of deriving the fluxes of the lensed QSO images is reduced to solving a system of ordinary linear equations. The shape and width of the light distribution of the PSF do not really matter. Our method can also be successfully used for images performed in the defocused mode [246,247].

*New light curves*. As a result of applying the adaptive PSF-fitting method to the Maidanak H1413+117 data set, we have provided new long-term photometric light curves of the lensed images. To determine the photometric error bars affecting each data point, we have simulated CCD images for each of these 500 frames including key factors such as: the Poisson noise due to the GLS, white noise for the sky background, uncertainties of the lensed image centring at the sub-pixel level, etc. The measurement of the error magnitudes of the lensed components corresponds to the standard deviation of the distribution of the results obtained for the various series of simulated frames.

*Brightness and colour variations.* We distinguished the light curves of the Clover Leaf lensed components in the V and R filters, and the V–R colour curve. The light curves of the whole system and of the individual components correlate very well with each other. We see two types of variations: short-term (with amplitude ~0.1 mag) during separate seasons and long-term (with an amplitude larger than 0.15 mag) over the timescale of several seasons. The colour changes are also characteristic of quasars (as in UM673, [234]) – with an apparent flux decrease leading to some reddening, and vice versa – during an apparent flux increase, the system becomes bluer.

*Time delay calculations*. Our long-term photometric light curves have allowed us to estimate the time delays between the lensed images of the Clover Leaf. In the calculations, we separately relied on the seasons between 2003 and 2006 showing the most frequently sampled data. We did not use the whole light curves because of gaps between seasons with durations of several months. On the other hand, microlensing events may change the magnitude difference between the components (this in turn makes it impossible to use the light curves of the components in a general way).

This problem was solved with the help of different methods and approaches: we used chi-squared tests with linear interpolation and two different dispersion formulas. To estimate the errors of our time delay predictions, we generated synthetic light curves on the basis of previously calculated magnitudes from simulated CCD frames. As a second alternative method, we used the classical method of normal distribution of stellar magnitudes. Also, we considered separately the DLC and MFLC.

Fortunately, thanks to a significant increase in flux of all the components in 2004 (see Figs 4.9 and 10;JD= 3130–3150), we could obtain some relevant values for the time delays. Our values for  $\Delta t_{AB}$  and  $\Delta t_{AC}$  are in full agreement with those obtained in [225], whereas we derived a time delay between the A and D components ( $\Delta t_{AD}$ ) that is not consistent with the previous one (using  $1\sigma$  confidence intervals). This discrepancy is likely due to an appreciable microlensing variation in the R-band light curve of the component D (see Sections 4.6 and 4.7).

*Micro-lensing events*. The complex and sometimes uncorrelated behaviours of the light and colour curves indicate some active intrinsic variability and continuous influence of micro-lensing. The micro-lensing rate measured for the A, B, C components, on average, corresponds to the characteristic rate of micro-lensing  $10^{-4}$  mag day<sup>-1</sup> reported for other known gravitationally lensed quasars [4,46,154,225,231].

However, we found an unusually strong influence of micro-lensing in the D component. In 2004, the light of this component has varied with a rate of  $10^{-3}$  mag day<sup>-1</sup> and almost reached the brightness of the C component in 2005. We can assert that this is a recurrent process, as similar flux variations have been previously reported [210,211].

Even more dramatic brightness changes of the lensed components due to micro-lensing have been reported for Q2237+0305 [48,248]. These findings support the view of Witt et al. (1995) about the general nature of quadrupole systems and the inevitability of micro-lensing events in them. Further evidence of the continuous influence of micro-lensing is the time variability of the magnitude difference between the lensed components, which vary over a very large range.

Also we were able to calculate possible mass for body caused microlensing in H1413+117 in 2004-2006 years. We found this value equal about giant planet mass. The same calculations for double GLQs gave us mass of normal stars.

Lens model and lens redshift. Following the lens scenario proposed in [225], we have modelled the H1413+117 lens system with an SIE (for the lens galaxy G1), an SIS (for the lens galaxy G2) and an external shear (for galaxy overdensities and other contributions). Based on available observational constraints (resp. the lensed image positions, mid-IR flux ratios and two sets of time delays derived in the present work), we have improved the lens modelling and updated the estimation for the lens redshift:  $z_l = 1.95^{+0.06}_{-0.10}$  for the  $\chi^2/MFLC/\delta = 30/\text{group}_I$  set of time delays. We found a slight improvement in the  $\chi^2_{\text{tot}}$  when we relax the time delay constraint on the component D but without significantly affecting the determination of the lens redshift value:  $z_l = 1.97^{+0.07}_{-0.11}$ . These two estimations are in good agreement with the previous one derived in [225]. Due to the galaxy spectrum Ca II break feature, the main lens galaxy detection in HST/NICMOS-2 images (IR *H*-band F<sub>160W</sub> and F<sub>180M</sub> filters), but not in HST/WFPC2 images (IR *I*-band F<sub>160W</sub> and F<sub>180M</sub> filters), seems to be more compatible with a high value for the lens redshift,  $z_l \sim 1.95$ , rather than  $z_l \sim 1$ .

As a recommendation, we think that the photometric adaptive PSF-fitting method which has been developed by us is very promising, since it can be successfully applied to a very wide class of CCD frames. This applies to images and CCD frames with different kinds of distortion.

We also provided accurate values of the time delays in H1413+117. On the other hand, due to the strong micro-lensing variability in the D component we should remain very careful about the use of the time delay values between the pairs of the AD, BD, CD components. It would be very nice to confirm these results from independent observations.

## CHAPTER V. THE WIDE FIELDS CASES: THE METHOD OF PROCESSING CCD IMAGES IN THE TDI-MODE

# §5.1. The 4-m International Liquid Mirror Telescope and its observational capabilities

The 4-m International Liquid Mirror Telescope (ILMT) project results from a collaboration between the Institute of Astrophysics and Geophysics (Liege University), the Canadian Astronomical Institutes from Quebec (Laval University), Montreral (University of Montreal), Toronto (University of Toronto and York University), Vancouver (University of British Columbia) and Victoria (University of Victoria) and the Aryabhatta Research Institute of Observational Sciences (ARIES, India). Several colleagues from the Royal Observatory of Belgium, the Poznan Observatory (Poland), the Ulugh Beg Astronomical Institute of the Uzbek Academy of Sciences and National University of Uzbekistan have also joined the project.

In the last two decades the telescoping technology, including optical has tremendous development. A large number of giant telescopes were put into operation: the world's largest single-piece mirror telescope – Large Binocular telescope (LBT) located on Mount Graham (Arizona, USA) from 2005 with two 8.4-m mirrors; at the same year the 10-m Southern African Large Telescope (SALT) designed mainly for spectroscopy and consisting of 91 hexagonal mirror segments each with a 1-metre inscribed diameter, resulting in a total hexagonal mirror of 11.1 m by 9.8 m gave its first light; in 2007, the Gran Telescopio Canarias (GTC) the Canary Islands (Spain) with a mirror diameter of 10.4 m consisting of 36 hexagonal segments began operating; in 2024, the 24.5-m Giant Magellan Telescope in Chile will be put into operation. There are many more such examples.

These telescopes are optical telescopes with different mounting systems. What unites them is that they use glass mirrors or their complexes as a lens. Before applying a layer of aluminum or silver to form a reflective surface, these mirrors require careful grinding and polishing, with an accuracy of several tens of nanometers. These large modern telescopes use sensor systems and mechanisms to actively control the shape of the mirror. This is called adaptive optics  $\phi$ TB counteracts the distortions created by the atmosphere. With these telescopes, astronomers have achieved impressive results, these tools represent the pinnacle of modern technology. But at the same time, they require huge financial costs (about \$10 million per telescope with a mirror diameter of 6 meters).

It is surprising that comparable accuracy can be achieved only by rotation of the container, covered by mercury. The surface of the liquid in equilibrium is the surface of constant potential energy. As a rule, the potential energy of an object is proportional to its height. Thus, the surfaces of most liquids are flat. But suppose the liquid rotates at a constant angular velocity around a vertical axis. In this case the potential energy of an arbitrarily small particle of the liquid has two components, one of which grows with height, and the other decreases with the square of the distance from the rotation axis. This leads to the fact that the liquid takes the form of a paraboloid. Rays parallel to the rotation axis are reflected so they converge in the focus of the mirror (Fig.5.1.). To correct rays from other directions, it is enough to add several corrective lenses [251-255].

The idea to use a rotating fluid to focus light is not new. The Italian astronomer E.Capocci was the first who described this possibility in 1850, although he did not implement the idea in practice. This idea was first implemented in an experiment in 1872, when Henry Skey created a liquid mirror with a diameter of 35 centimeters [see [249]). In 1909 R. Wood first built a telescope with a liquid mirror. It rotated on a mechanical bearing and was connected to the engine via a drive belt. With this telescope, Wood was able to obtain resolved images of a close four-component system in the Lyra, which angular size is 2.3 arcseconds. These results were impressive.

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However, the Wood's telescope was not so practical. It suffered by vibrations and a small but noticeable swing of the mirror. Moreover, the inaccuracy of the control of rotation speed has led to fluctuations of the focal length of the mirror. Since the rotation axis is vertical, the telescope could only observe a small area of the sky directly above the head. These shortcomings outweighed the achievements of the telescope, which explains the lack of interest of observers to the idea of using the liquid mirror in astronomy for a long time.

This idea was returned in the early 1980s, when E. Borra et al. reviewed the pioneer concept of R. Wood and came to the conclusion that modern technologies can solve the technological problems that have suspended the previous project [250, 281]. The first large fully operational LM Telescope has been the Large Zenithal Telescope (LZT) built by Prof. Hickson [251, 252, 253, 254, 255]. The technology of LMs is relatively simple. Three components are required: i) a dish containing a reflecting liquid metal (essentially Hg), ii) an air bearing on which the LM sits, and iii) a drive system.



**Fig.5.1.** Principle of a liquid mirror telescope. The primary mirror is a rotating reflective fluid, taking a parabolic shape under the combined action of the gravity and the centrifugal force. A camera inserted in the focal plane of the paraboloid will image the field-of-view near the Zenith [280].

The operation scheme of an liquid mirror telescope is described in detail in [280]. A liquid surface at equilibrium is perpendicular to the vector of the force it experiences. The centrifugal force is  $\omega^2 r$ , where  $\omega$  – paraboloid-dish angular

velocity, r – distance to the rotating axis. The gravity is assumed to be constant all over the surface. Set the angle  $\theta$  between the liquid surface and the horizon as:

$$\tan \theta = \frac{dz}{dr} = \frac{\omega^2 r}{g} \tag{5.1}$$

Integrating (5.1) and setting the origin of the z axis at the liquid surface central height, we obtain the shape of the liquid surface

$$z = \frac{\omega^2 r^2}{2g}.$$
(5.2)

Comparing (5.2) with the general equation of a parabola with a focal length F:

$$z = \frac{r^2}{4F} , \qquad (5.3)$$

we see that the rotating liquid surface is a paraboloid having a focal length

$$F = \frac{g}{2\omega^2} \,. \tag{5.4}$$

A rotating dish containing a reflective liquid thus acts as a parabolic mirror. Its focal length F is determined by the angular velocity through (5.4).

Parabolic mirrors have the property that all light rays incident parallel to the axis of symmetry of the paraboloid are refelected towards its focal point, as shown in Fig. 5.1. Consequently, inserting a camera or a sensor at the focal point of the paraboloid allows to image the field-of-view at the zenith of the mirror. Thus, a liquid mirror telescope is obtained by using a reflective liquid as the primary mirror and by inserting a sensor on the focal surface. The angular field-of-view (FOV) of the telescope is then determined by the focal length F and d - the size of the sensor:

$$FOV = \frac{d}{F}.$$
(5.5)

From the described properties of the liquid mirror telescope, follows it has an azimuthal mount and looks directly at the zenith. Because the local gravity is used to generate the primary mirror shape, it is not possible to incline it in order to point or track an object, as it is done with normal telescopes. Furthermore, because of the earth rotation, all objects in the telescope FOV are moving at the sidereal rate. It is thus impossible to image the near-Zenith FOV using conventional integration techniques as stars would leave tracks due to their diurnal motion over the detector.

The rotation of the Earth induces the motion of the sky across the detector surface. The CCD detector works in the Time Delayed Integration (TDI) mode, i.e. it tracks the stars by electronically stepping the relevant charges at the same rate as the target drifts across the detector, allowing an integration as long as the target remains inside the detector area. The CCD detector is cooled down to a temperature near -100 °C, in order to reduce as much as possible the dark current. The complete telescope is protected by a building with a sliding shutter for opening the roof. The building is air-conditioned and is also equipped with four exhaust fans.

Using a CCD camera with TDI imaging, the integration time t is fixed by the time necessary for the objects to go through the telescope *FOV*. Defining the sidereal rate *SR* of an object at the telescope zenith, the integration time is

$$t = \frac{FOV}{SR}.$$
(5.6)

The mercury mirror of the ILMT has a 4 meter diameter with an aperture of f/2 defined by the speed of rotation. A 4Kx4K CCD camera manufactured by 'Spectral Instruments' and which can operate over the 4000 to 11000 Å spectral range (SDSS filters g', r', i' are available), will be positioned at the prime focus of the ILMT at about 8 meters above the mirror. The mirror being parabolic in shape requires an optical corrector to get a flat focal surface of about 24' in diameter. All these elements are mechanically coupled by an external structure and a spider.

The ILMT equipment had been transported from Belgium to Devasthal in December 2011. However due to unexpected delays in the construction of the ILMT building, the erection of the telescope by AMOS (Advanced Mechanical and Optical Systems, Belgium) could only start in March 2017. One compressor is needed to operate the air bearing. However, in order to avoid any interruption of the mirror rotation (cf. during the maintenance of the compressed air system), it is best to have two parallel air systems. Therefore, we have decided to purchase two air compressors and two air tanks that will be working independently. If one air system fails or needs to be serviced, the second one will be automatically switched on. The two air compressors and air tanks have been installed by the Gardner-Denver company located in Delhi. In addition to the pneumatic air control system provided by AMOS (see Fig.5.2), we have improved this by adding additional valves, air dryers, air filters and sensor (pressure, temperature, humidity and dewpoint) modules along the two independent and parallel air systems to make sure that the air bearing will be operating under optimal and very safe conditions.



**Fig. 5.2.** The AMOS pneumatic air control system installed in the ILMT main building.

On 2nd of March 2017, the ARIES and AMOS teams have proceeded with the transportation of the container in which the ILMT equipment had been stored in Devasthal since February 2012. One can see in Fig. 5.3. the ILMT equipment (telescope structure, air bearing, optical corrector) being unpacked. Some rust was found on various pieces of equipment but it has been successfully removed and cleaned by the AMOS team after two days of intensive work.



Fig. 5.3. Big blue container with the ILMT equipment just installed in front of the compressor and ILMT buildings.



5.4. Fig. The optical adjusting devices, has been lifted and mounted on the prime focus holder



corrector, Fig. 5.5. Positioning and centering of the assembled with the tip-tilt and focus air bearing just below the optical corrector. The air-bearing has been adjusted horizontally by means of three screws grounded to the floor.

During the period 4-10 March 2017, the AMOS, ARIES and Liege University teams have contributed to the erection of the ILMT structure. We have achieved the progress in installation of the four inferior pillars, the lateral reinforcing bars, the four superior pillars and the spider structure to hold the optical corrector.

We may also see the presence of a deployable platform to access the prime focus of the telescope. After unpacking the optical corrector, the latter has been assembled with a tip-tilt and focus adjusting device. This whole system has subsequently been mounted on the structure of the prime focus holder (see Fig. 5.4.). The AMOS team members have then precisely positioned on 16 March 2017 the air bearing just below the center of the optical corrector. The air bearing has then been chemically and firmly anchored inside the concrete of the central pier (see Fig. 5.5.). It has been adjusted by means of three screws belonging to the three-point mount that aligns the axis of rotation parallel to the gravitational field of the Earth. On 17 March 2017, the mirror has been set on the air bearing. This light and rigid mirror consists of a foam core sandwiched between carbon fiber sheets. Two layers of polyurethane have subsequently been cast while the recipient was spinning around its vertical axis. Its shape is thus that of a paraboloid with a focal length near 8m. The construction of this mirror took place at AMOS in 2009.

A nice view of the whole ILMT can be seen in Figs. 5.6. The first photograph was taken from the top of the roof of the ILMT building. We clearly see the optical corrector projected on the primary mirror. An interface between the corrector and the CCD camera still needs to be installed as well as the CCD camera itself. Let us also note that during observations with a liquid mirror, no mercury vapors are generated after some time (typically 8 hours) because the surface of mercury gets covered with a thin layer of mercury oxide that prevents any further evaporation. Finally, Fig. 5.7. shows a view on the compressor (front), the control (middle) and the main ILMT (back) buildings rather than Figs. 21-24 present aerial views of the ARIES Devasthal Observatory around the ILMT site.

Given the zenith observing mode of a liquid mirror telescope and in order to access the northern galactic pole, the Devasthal observatory is ideally located in latitude (near +29° 22' 26"). From this site, a deep (i' = 22 mag) survey will approximately cover 90 square degrees at high galactic latitude, which is very useful for gravitational lensing studies as well as for the identification of various

classes of interesting extragalactic objects (cf. supernovae, clusters, etc.). Preliminary estimates of the photometric and astrometric accuracies achievable with the ILMT as a function of the magnitude of point-like objects may be found in the PhD thesis of Brajesh Kumar (2014, unpublished but accessible via the URL: http://orbi.ulg.ac.be/handle/2268/174851).





top of the roof. The optical corrector is control and main ILMT buildings. superimposed over the mirror. On the top is visible the folding platform.

Fig. 5.6. View of the ILMT from the Fig. 5.7. View of the ILMT compressor,

Liquid mirror telescopes (hereafter LMTs) cannot be tilted and hence cannot track the way conventional telescopes do. As written above, to track with imagery, one relies on the TDI technique, also known as drift scan, that uses a CCD detector that tracks by electronically stepping its pixels. The information is stored on disk and the night observations can be co-added with a computer, resulting into longer integration time images. Conversely, the image resulting from the subtraction of a nightly recorded image by a reference image characterized by a high S/N may

easily lead to the identification of photometrically and/or astrometrically variable objects, including variable point-like components superimposed on extended objects (cf. multiply imaged quasars, supernovae, etc.). The collected data will be ideally suited to perform a deep photometric and astrometric variability survey over a period of typically 5 years. A pipeline to digest all those data is being set at the Poznan Observatory.



**Fig. 5.8.** All sky image centered on the galactic center in the Hammer-Aitoff projection. The strip of sky accessible by the ILMT is superimposed over the image. Equatorial coordinate isocurves are projected as grey lines on the all sky image, NP and SP being the North and South equatorial pole, respectively [280].

Some research projects (e.g. time consuming surveys, long term photometric monitoring programs) simply cannot be envisioned with classical telescopes but become possible with a dedicated liquid mirror telescope. This is particularly true for the types of research where the specific region of the observed sky is not very important (e.g. cosmology). Furthermore, the quality of the recorded observations is optimal at zenith since both the seeing and the transparency are the best there, at all times. Moreover, the observing efficiency is very high. Indeed, when observing

each night the same strip of sky, there is no time lost for slewing, field acquisition, readout times, etc.

Let us finally note that flat fielding and CCD image de-fringing are much more accurate than during classical observations since the images are actually formed by averaging the signal over entire CCD columns (in the direction of the scan). Among the apparent disadvantages, let us recall that one can only observe at zenith a strip of constant declination. At the latitude of  $+29^{\circ}22'26''$ , a band of approximately half a degree covers 156 square degrees, with 88 square degrees being covered at high galactic latitude ( $|bII| > 30^{\circ}$ ). The nightly integration times are rather short, typically ~100 sec. but as already stated before it is always possible to co-add data from selected nights in order to get longer integration times. The volume of collected data is estimated to be ~ 10 Gby per night.

The advantages and limitations of this unusual type of telescope constrain the science that is feasible and thus the science requirements. The ILMT will be entirely dedicated to projects of high scientific interest dealing with astrometric or photometric variability. They concern galactic and extragalactic astrophysics. A short list of the science drivers namely includes:

i) the statistical determination of the cosmological parameters  $H_0$ ,  $\Omega_M$  and  $\Omega_{\Lambda}$  based upon surveys for multiply imaged quasars which consist of compact gravitational lens systems,

ii) the statistical determination of these same cosmological parameters based upon surveys for supernovae,

iii) a search for quasars and observational studies of large scale structures, iv) the determination of trigonometric parallaxes of faint nearby objects (e.g. faint red, white, brown dwarfs, halo stars and other very low mass stars, etc.), v) the detection of high stellar proper motions to probe a new range of small scale kinematics (stars, trans-neptunian objects, etc.),

vi) a wide range of photometric variability studies (cf. photometry of stars, RR Lyrae, transiting extra-solar planets, novae, supernovae, micro-lensing and

other transient events, photometry of variable AGN over day to year time scales, etc.),

vii) the detection of low surface brightness and star-forming galaxies, and other faint extended objects (galactic nebulae, supernova remnants, etc.),

viii) serendipitous phenomena, and finally, ix) the production of a unique database for follow up studies with the 3.6m Devasthal Optical Telescope (DOT). Follow up studies with other large telescopes (cf. VLT, Gemini, Keck, GranTecan, SALT, etc.) will also be encouraged.

More details on the 4-m ILMT project and its scientific cases may be found in [256,257,258,280], and via the following web site: http://www.ilmt.ulg.ac.be

### §5.2. The 4-m ILMT data reduction pipeline

The 4-m ILMT will continuously scan a  $\sim 0.5^{\circ}$  band of sky in the TDI mode, also known as drift scan, where the telescope cannot be tilted and hence cannot track the way conventional telescopes do [258,259]. In this particular technique, the CCD detector tracks by electronically stepping its pixels and gigabytes of information will be stored on disk during each night of observation. This huge amount of data must be reduced and analyzed in a timely fashion and it is clearly necessary to automatize as much as possible these operations in order to increase the speed at which they can be carried out. The existing general-purpose astronomical software packages are not sufficiently efficient to perform such tasks. The 4-m ILMT data reduction pipeline has been designed specifically to carry out the reduction of the TDI data and to perform their astrometric and photometric calibrations.

In order to prepare ourselves for when the ILMT will become operational, a replica of direct ILMT observations has been carried out using the ARIES 1.3 m optical telescope at Devasthal, India. In this paper, we present the optimal algorithms adopted to reduce these science images. An interesting application of this software is to contribute to the statistical awareness on space environment

around the Earth. In this context, we present the detection and characterization of 9 space debris. Most of these appear during the TDI observations carried out at dawn and dusk.

Each image obtained with the Devasthal 1.3-m telescope (May-Jube 2013 and 2014) equipped with a direct CCD camera ( $2K \times 2K$ ) working in the TDI mode consists of a long strip of sky covering a field of view of nearly 2.3° in right ascension and 0.15° in declination. During observations



**Fig. 5.9.** A typical images obtained at the 1.3-m telescope at the Devasthal observatory. The original image is  $30000 \times 2048$  pixels. At bottom are zoomed areas of the image by size of ~6000× 2048 pixels.

In the course of observations in the TDI mode, CCD images of  $3000 \times 2048$  pixels were obtained (Fig. 5.9). In the top is general view of the image, and bottom is different areas of that image. Note that the CCD is oriented so the horizontal axis corresponds to the RA, and the vertical axis to the declination in the Equatorial coordinates. And each frame of images has a length of about 11 minutes

on a straight ascent, or the same, in time. This means the images were obtained with an exposure equivalent to the specified time.

The data reduction procedure includes the following steps: dark subtraction, flat-field correction, cosmic ray removal, sky subtraction, astrometric and photometric calibration. First of all, it is necessary to subtract the dark current and normalize by flat field for images. It is necessary to find the average slices on two axes for each image of the dark current. At the same time, if we want to find the average value of the dark current along the vertical axis, it is necessary to carry out averaging on all pixels of the horizontal axis, and vice versa. Results of this procedure given in Fig. 5.10.

In our case the science images show a systematic variation in the gradient of the dark current along the declination direction as a function of the changing ambient temperature during the night. To correct for this effect, we have first calculated the average of the pixels along the right ascension axis for each dark frame so that we obtain a 1 dimensional 2K (2048) file of pixel values along the declination axis. The plot shown in Fig. 5.11 illustrates the different slopes derived of a straight line fit to the above mentioned pixel values for different temperatures. It is clearly evident that there is a change in the slope of the straight lines as a function of the CCD temperature. So, we have developed appropriate algorithms to subtract at best the relevant dark current.

In the dark subtracted science images, the average of the pixel values along the right ascension axis with sigma-clipping is taken in order to only include those pixels enlightened by the sky so that we avoid all pixels illuminated by stars, cosmic rays and exclude the effects of bad pixels. We get 2048 pixel values along the declination axis which can be treated as a 1D flat. Since the star images are trailed along the rows of the CCD, the pixel sensitivity issue becomes a row (1D) sensitivity one. And the pixel-non-uniformity is corrected by dividing the dark subtracted frame by the normalized 1D flat throughout all the columns. An

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example of CCD frame before and after the flat field correction is illustrated in Fig. 5.9-5.10.



**Fig. 5.10.** The upper and lower images show an example of CCD frame before and after flat field correction.



**Fig. 5.11.** Varying behavior of the dark current as a function of the CCD temperature



**Fig. 5.12.** Variation of the sky background as a function of time during a single night of observation. The plot in blue shows the sky flux in ADU and the thick black line shows the estimated sky level, also in ADU.

During a continuous and long TDI observation, the sky may vary throughout the night due to the presence of the moon, clouds, dusk light and/or twilight. The numerical approach implemented in the sky background estimation consists of an optimized combination of "sliding average method" and "two dimensional polynomial fit" along the right ascension axis which shows the trend in variation of the sky level as time goes. The variation of the sky brightness during a single night and the estimated sky value are shown in Fig. 5.12.

The result of observations is a two-dimensional frame of size  $M \times N$  pixels. The CCD-camera is oriented so the vertical axis of the frame corresponds to the declination  $\delta$ , and the horizontal right ascension  $\alpha$  in the equatorial coordinate system. Each element of the frame (i, j) corresponds to a certain value of photoelectrons  $I(\alpha, \delta) = I(i, j)$ , registered by the CCD detector. The analysis of raw frames showed that the background on the frames is heterogeneous both in the directions of the axes and from frame to frame.

Figure 5.12 shows the averaged slices of raw frames obtained at different times along the vertical and horizontal axes, respectively. Each curve is obtained by averaging along the given axes:

$$\overline{I(i')} = mean[I(i_{fixed}, j)]$$

$$\overline{I(j')} = mean[I(i, j_{fixed})]$$
(5.7)

Complex and sometimes unpredictable behavior of the sky background can be caused by a number of factors. In classical cases, CCD images come simply – the average value of the sky background over its entire area is subtracted from the frame. In our case, such an approach would be incorrect – as can be seen from the figures, in each frame the background component is different from the other complex picture, a simple subtraction of the average value will not lead to anything.

To solve this problem, we used a step-by-step algorithm that will allow us to take into account the local variations of the sky background as accurately as possible:

- In the initial frame, we obtain background curve along the declination axis as a one-dimensional vertical matrix I(i') of size N by averaging I(i, j) in each row.
- 2. Each column of the original image is divided by I(i'), thus obtaining a

normalized frame I'(i, j).

- Next, in the new frame with the intensity I'(i, j) we get the brightness curve along the axis of right ascension in the form of a horizontal matrix I(j') of size M by averaging I(i, j) in each column.
- 4. In the procedures 1 and 3, the pixels for which  $I(i,j) = \overline{I(i,j)} \pm 3\sigma$ should be excluded from statistical processing, where  $\overline{I(i,j)}$  is the average intensity over the frame,  $\sigma$  is the standard deviation. This is due to the presence of bright objects with extensive halos, which leads to overestimated estimates of I(i') and I(j').
- 5. We obtain a smoothed vector I(j'') for I(j') by its polynomial fitting or averaging with a floating window.
- 6. We obtain a two-dimensional frame for the background substrate I'(i,j)(see Fig.5.13) of size  $M \times N$  by vector product I(i') and I(j'').
- 7. Subtract I'(i,j) from I(i,j) and as a result we have a wide field frame cleaned of the sky background taking into account its local features.



Fig. 5.13. Sky background pattern (bottom), and cleaned frame (up).



**Fig. 5.14.** The average frame slices, with cleaned sky background. Up: intensity change along the vertical axis (declination). Bottom: the same for the horizontal axis (right ascension – time). The intensity is given in units of ADU.

We have developed a computer program, which gradually leads to the execution of all these procedures. Figure 5.14 shows the averaged slices of the resulting frames, along the vertical and horizontal axes, respectively. It can be seen that the resulting slices are now concentrated near zero and constant along the entire length of the axes. Now the received frame is ready for the next stage – direct photometry of the objects registered by the detector.

### §5.3. Astrometric and photometric calibration problems

Here we consider the problems associated to astrometric and photometric calibration of images obtained in TDI mode within the framework of the ILMT project. As we mentioned this telescope scans the sky area at the zenith with a width of approximately 0.5 degrees. To do it we had a number of wide field images obtained at the Devasthal Observatory in May-June 2015 by the 1.3 meter telescope. This telescope was directed to the zenith and gave us wide field images type, with which we will work soon after the ILMT will start to work.



**Fig.5.15.** Variation of the central declination during time of the observations. Sinus curve – coordinates in the standard epoch of J2000. Horizon curve –coordinates reduced to observation epoch.

First, let's focus on astrometric calibration. The fact that in the observations we are operating with a right ascension  $\alpha_T$  and declination  $\delta_T$ , which correspond to time of this observations - *T* (observations epoch). Astrometric calibration gives us the coefficients of transition of the observed coordinates in the standard coordinate system J2000  $\alpha_{2000}$  and  $\delta_{2000}$ .

First of all it became clear that the right ascension and declination are constantly varying and the reason for this is the precession of the Earth. To test this statement, we identify the centers of the images using ALADIN data base and got the dependence of  $\alpha_{2000,C}$  versus  $\delta_{2000,C}$  (red curve in fig 5.15), which is sinus-like.

$$\alpha_{T,C} = \alpha_{2000,C} - (3.075^{s} + 1.336^{s} \sin \alpha_{2000,C} \tan \delta_{2000,C}) * (2000 - T)$$
  
$$\delta_{T,C} = \delta_{2000,C} - 20.043'' \cos \delta_{2000,C} * (2000 - T)$$
(5.8)

Using the precession formula (5.8), we reduced the coordinates of the centers to the epoch 2000 and obtained a smooth line (blue line, Fig.5.15). This behavior of the equatorial coordinates showed it is necessary to consider the effect of Earth's precession in the coordinate transformations.

To determine the right ascension  $RA_T$  and declination  $DEC_T$  of objects registered by the CCD detector in wide fields images, we need to know the star time ST of the first column in the images and the geographic latitude  $\varphi$  of the telescope site. The sidereal time of the initial column is continuously measured in the course of observations and recorded in the name of the corresponding file. Since the telescope is directed to the zenith, the observed declination will correspond to the geographic latitude of the observation site, and the right ascension to sideric time. The geographical latitude of the astronomical site is  $\varphi = \delta_{C,T} = 29^{\circ}21'42''$ . In addition, to fast determine the pixel coordinates x and Y of the objects registered in the CCD images, it can use the well-known program Sourse Extactor. Then the equatorial coordinates of the objects in the observed epoch (*T*) can be obtained through the formulas

$$RA_T = ST + X * pix_X$$
$$DEC_T = \delta_{\text{центр}} + (1024 - Y) * pix_y$$
(5.9)

where  $pix_x$  and  $pix_y$  – are pixel scales along horizontal and vertical axis, respectively. In the first approximation, we took these scales equal to 0.4" per pixel. Now it was necessary to transform these coordinates into the epoch of 2000 and compare with the catalog data GAIA. For this transmission we used the same formula of precession:

$$RA_{2000} = RA_T + (3.075^S + 1.336^S \sin RA_T \tan DEC_T) * (2000 - T)$$
  
$$DEC_{2000} = DEC_T + 20.043'' \cos DEC_T * (2000 - T)$$
(5.10)


**Fig. 5.16.** An example of the difference between cataloged equatorial coordinates and computed one. The calculations were carried out with the pixel scale 0.4".



**Fig. 5.17.** Average difference between catalogue equatorial coordinates and calculated for all frames in two observation days.

Now compare  $RA_{2000}$  and  $DEC_{2000}$ , calculated by the formula (5.10) with the catalog coordinates  $\alpha_{00}$  and  $\delta_{00}$ , and the difference between them is showed in the fig. 5.16 and fig/5.17. As you can see, the result was unsatisfactory, otherwise we had to get a flat line, the points of which are randomly scattered about zero. So, we found  $RA_{2000}$  and  $DEC_{2000}$  do not correspond to the catalog values.



**Fig. 5.18.** Examples of the variation of the angular pixel scale on two axes over time for two days are shown.



**Fig. 5. 19.** An example of the difference between the catalog equatorial coordinates and calculated ones based on the real pixel scale.

The reason, of course, is the continuous change of the angular pixel scale. To determine this parameter, we used the golden section method and obtained graphs of the pixel scale versus time (Fig. 5.18).



**Fig. 5.20.** Photometric calibration: the calibrated magnitude of each source can be found using the linear relation established between the instrumental and standard magnitudes. On the left – the "main sequence" stars with indication of the calibration coefficient, on the right – the histogram of difference between the instrumental and standard magnitudes.



**Fig. 5.21.** The dependence between differences of standard and instrumental magnitudes versus the SDSS-DR9 database.

Now, if the formula (5.9) substitute the real values of the pixel scale, which vary constantly in time, we get a very good convergence of coordinates. This is shown in the fig. 5.19, which shows the same coordinate differences as in the fig. 5.16. It can be seen that taking into account the variability of the pixel scale gives very good results.

And now let's consider the issue of photometric calibration. In this case, we will compare the magnitudes obtained from observations with data from the SDSS database (Fig. 5.20). For photometric calibration, we will need photometric reference stars. These objects should not be variable, and therefore they should be on the "main sequence" in the graph of the dependence of the instrumental magnitude versus standard magnitudes (see. Fig. 5.20, left). The difference in magnitudes of these objects, obtained in two systems (instrumental and standard), will give us a calibration coefficient. In the fig. 5.21 the desired dependence for one frame with different sizes of aperture photometry -10,15,20 pixels are shown, respectively. As you can see, the larger the aperture of photometry, the more accurately magnitudes are calculated.

## **§5.4.** Summary and discussion on Chapter V

For zenith-pointing observations, LMTs can deliver the same performance as classical telescopes, with much lower cost and greater simplicity of operation. Although several liquid mirror telescopes (e.g., the 2.7-m and the 6-m University of British Columbia and NODO LMTs) have been previously used for sky observations (cf. space debris, earth atmosphere, sodium atmospheric layer), they were first-generation instruments not optimized for astronomy, nor located at highquality astronomical sites [259]. The ILMT has essentially been developed for celestial observations from a good astronomical site.

It is shown that the most important effects in astrometrical calibration are, firstly, the precession of the Earth, which systematically changes the equatorial coordinates relative to the standard epoch, and secondly, the random change in the pixels angular scale. The latter is most likely due to the influence of temperature on the main mirror, which changes the focal length of the telescope.

The 4-m ILMT is an instrument that will be entirely dedicated to a photometric and astrometric variability survey of a narrow strip of sky (~ half a degree) passing through the zenith. Each night, a long CCD image recorded under the best image conditions (atmospheric seeing and extinction) will be compared to a reference one and any transient source or highly variable object should be easily detected. Thanks to the proximity of the 3.6m DOT, follow-up strategies should be easily implemented.

## MAIN RESULTS AND CONCLUSION

According to the results of the research carried out on the theme of the doctoral (DSc) dissertation "CCD-photometry of selected gravitationally lensed quasars and wide field cases" the following conclusions are drawn:

- 1. Monitoring observations of the objects of research were carried out. For selected double and quadrupole component objects the noted CCD observations were processed, the light curves in the V, R and I filters have been obtained and investigated. They showed the sources-quasars of all considered objects continuously vary their brightness, and for double GLQ limit of variation on average  $\Delta m \sim 0.5$  mag, and for quadrupole GLQ  $\Delta m \sim 0.3$  mag, which is significantly less than the previous type. The brightness variations of the lensed components are related to non-stationarity of the source (bursts, irregularity of the accretion process, etc.) and microlensing.
- 2. Two types of microlensing were determined: depending on the rate of brightness variation they can be background (mag/day) and strong (mag/day); and depending on the duration they can be long-term (more than a year) and short-term (up to a year). Our results on microlensing support the hypothesis of H. Witt, according to which microlensing in quadrupole GLQ has general nature and the inevitably in them.
- 3. The effect of long-term microlensing lasting more than 10 years was found for the first time in the GLQ SBS1520+530. It was also found that the quasar undergoes periodic brightness variations of small amplitude.
- 4. For the first time in GLQ FBQ0951+263 it was shown that, as in the previous case, exactly the long-term microlensing dominates over short-term ones. The microlensing in this system has been going on for more than 10 years, and there is a process of decline of the microlensing effect on the component B.
- 5. The light curves of the A1, A2, B and C components in the GLQ PG1115+080 allowed to calculate the time delays between three pairs of components:  $\Delta t_{AB} = 4.4^{+3.2}_{-2.4}$  days,  $\Delta t_{AC} = 12.0^{+2.4}_{-2.0}$  days and  $\Delta t_{BC} = 16.4^{+3.4}_{-2.4}$  days.

- 6. The internal variability of the quasar-source with amplitude of about 0.4 mag is found. It is shown that the flux ratio between components A1/A2 varies not only in time, but also decreases with increasing wavelength of the filter. The microlensing in the images A1 and A2 in the period of 2001-2006 with the descending phase A1 and phase A2 of ascending also was discovered. The dependence of the (V-I) color index of components both versus the R magnitude and versus time is found. As the brightness of the quasar decreases, the color of the components shifts to the red side.
- 7. The new method of adaptive PSF fitting for photometric processing of the point images in dense fields has been developed and tested on the example of images of GLQ H1413+117. At the same time, an independent method for estimating the errors of photometric measurements based on the simulation of artificial images taking into account the noise and errors inherent in digital images has been proposed.
- 8. The light curves of the H1413+117 components showed signs of both internal variability and strong influence of microlensing with an amplitude of ~0.15 mag and a rate of ~10<sup>-3</sup> mag/day. An estimate of the lower limit for the body mass responsible for this microlensing was obtained (~10 M<sub>Jup</sub>).
- 9. The time delay values between the lensed components in GLQ H1413+117 have been calculated:  $\Delta t_{AB} = -17.4 \pm 2.1$  days,  $\Delta t_{AC} = -18.9 \pm 2.8$  days and  $\Delta t_{AD} = 28.8 \pm 0.7$  days (where A, B and C are leading, and D is the trailing component). A new method of generation of the synthetic light curves for estimation of time delay measurement errors is proposed.
- 10. Taking into account the values of time delay and on the basis of gravitational lens model in the H1413+117 with singular isothermal ellipsoid and an external shear the most probable value of its red-shift was obtained:  $z_l = 1.9^{+0.07}_{-0.11}$ .
- 11. For the first time the long-term (over 6 years) brightness variation in the source-quasar of the GLQ B1422+231 with amplitude of about 0.25 mag has

been found. Light curves of the three components also show microlensing. Variations in the color indexes of the components also confirm this statement. The proposed new values of time delays are:  $\Delta t_{AB} = 2.5 \pm 2.3$  days,  $\Delta t_{AC} = -7 \pm 1.5$  days and  $\Delta t_{AD} = 6 \pm 1.8$  days (A component is leading).

12. A computer program for calculation of the background component in the images obtained in the TDI mode has been developed. The precession effect was found in the images obtained during zenithal observations. It is shown that the astrometric calibration of the equatorial coordinates of objects requires taking into account the variable angular scale of pixels. It is found that the linear correction coefficient of the conversion to the standard value is sufficient for photometric calibration.

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