# ACADEMY OF SCIENCE REPUBLIC OF UZBEKISTAN INSTITUTE OF NUCLEAR PHYSICS 

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## DYNAMICS OF INTERACTION IN HEAVY IONS COLLISIONS AT THE ENERGY NEAR TO COULOMB BARRIER

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## DISSERTATION

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## Introduction

Topicality and relevance of the theme of the dissertation. At present, the world successes in appearance of the precise modern experimental facilities stimulate theoretical studies of nuclear structure and its role in reaction mechanism at heavy ion collision reactions. The study of reaction mechanisms based on the experimental data shows a decisive role of the entrance channel (initial stage) in the yield of products at energies of the around and above Coulomb barrier. The effect of entrance channel can be studied by analysis of products of reactions with different mass asymmetries of projectile-target pair, leading to the same compound nucleus. The study of the dynamics and mass (charge) distribution in collisions is one of the urgent tasks of modern experimenters and theorists.

To date, research centers around the world with heavy ion accelerators are conducting research to produce new isotopes of transuranic and superheavy elements. A significant difficulty in producing the heaviest elements in fusion evaporation reactions is exceedingly small cross sections, which can be attributed to the hindrance at the complete fusion of colliding nuclei and/or to instability of the being formed compound nucleus against fission. In addition, there is ambiguity in the estimate of the cross section for pure complete fusion, from the measured data, since the contribution of the products of the competing reaction channel, as quasifission is included in the fusion-fission cross section. The high accuracy of measurements of the ER cross section stimulates studies of the peculiarities of the reaction mechanism in experiments producing transuranic elements. However, one of the most interesting results of recent experiments in heavy ion physics is the discovery of a new type of reaction.

During the years of independence of our country, the science has been developed by providing theoretical and experimental investigations on nuclei fission and events to solve fundamental problems in the world, and defined results have been reached. Increasing the efficiency of nuclear technology area through the
application of innovative technologies of theoretical and applied investigations in the area of nuclear reactions with heavy nuclei has significant meaning in the Strategy of Actions on Further Development of Uzbekistan.

This research corresponds to the tasks stipulated in governmental regulatory documents and Decree of the President of the Republic of Uzbekistan No.PD-4512 "On works of further development of alternative energy sources" of 1 March 2013, Decree No.PD-4958 "On further improvement of the post-university education system" of 16 February 2017, Resolution No.PR-2789 "On measures of further improvement of the activities of the Academy of Sciences, organization, management and financing of scientific research works" of 17 February 2017 and Decree No.PD-4947 "On the Strategy of Actions on Further Development of the Republic of Uzbekistan for 2017-2021" of 7 February 2017.

Relevance of the research to the priority areas of science and technology development of the Republic of Uzbekistan. The dissertation research was carried out in accordance with the priority areas of science and technology development of the Republic of Uzbekistan: II. "Power, energy and resource saving".

Degree of study of the problem. Nowadays, many scientific groups in the world, such as Russian scientists (G.G. Adamian, N.V. Antonenko, R. V. Jolos, E. M. Kozulin), German scientists (J. Khuyagbaatar, S. Hofmann, W. Scheid), Italian scientists (G. Fazio, G. Giardina, G. Mandaglio), Korean scientists (K. Kim, Y. Kim, Yongseok O.), Chinese scientists (F. Zhang, P. Wen, L. Zhu), US scientists (E. Henry, W. U. Schröder), Indian scientists (M. Thakur, A. Shamlath) and others, make theoretical and experimental investigations to study the dynamics of collisions between heavy ions. However, such Uzbek scientists as V.P. Pikul, Yu.N. Koblik, A.K. Nasirov, R. B. Tashkhodjaev and others have many works on researching of properties of heavy ion, fission products, yield, kinetic energy and angular distribution of the reaction.

As a result of these studies, to obtain more accurate theoretical calculations in reactions between heavy ions, the dinuclear system (DNS) model have been developed by scientists from the Joint Institute for Nuclear Research (Russia) and
the Institute of Nuclear Physics (Uzbekistan). One of the advantages of dinuclear system model as compared with other models is that, it allows take into account the effect of the nuclear shell structure in theoretical studies of processes in collisions of heavy ions. There is a phenomenological model developed by Chinese theorists for calculating the fusion of massive nuclei, where the parameters are found by fitting the theoretical fusion cross sections to known experimental data.

Unfortunately, these methods are less informative for understanding the physics of nuclear fusion mechanisms. In addition, the contribution of the entrance channel to the reaction mechanisms and multi-nucleon transfer during the formation of a compound nucleus has not been studied until now.

Connection of the theme of dissertation with the scientific researches of the higher educational institution, where the dissertation was conducted. The dissertation was carried out within the framework of the scientific projects of the Institute of Nuclear Physics: FA-F2-F055 "A study of the reaction yield with heavy ions and nuclear fission" (2007-2011), FA-F2-F115 "Investigation of reaction mechanisms of multinucleon transmissions and fusion-fission of nuclei" (20122016), OT-F2-14 "Investigation of collective and microscopic properties of strongly interacting many-particle quantum systems" (2017-2020).

The aim of the research is devoted to the estimation of contribution of various reaction mechanisms to the observed yield of reaction products at different initial values of energy and orbital angular momentum.

## The tasks of the research:

calculation of total interaction potential of dinuclear system for different initial condition of angular momentum and orientation angles of colliding nuclei;
solving the equation of motion and obtaining an equation for estimating the contribution of subbarrier capture processes with different initial conditions for reactions ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ and ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$;
calculating mass and charge distribution of reaction products for nonequilibrium initial stage of the dinuclear system evolution for ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ and ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reactions;
computing the fusion, quasifission, fast fission and evaporation recidue cross sections in evolution of dinuclear system for the reactions ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ and ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$;
estimation of the contribution of the products of quasifission, fast fission, and fusion-fission to the mass distribution for the reaction ${ }^{48} \mathrm{Ti}+{ }^{208} \mathrm{~Pb}$
studying the dependence of charge distribution (yield of products) on different values of angular momentum for the reaction ${ }^{78} \mathrm{Kr}+{ }^{40} \mathrm{Ca}$;
analysing results of the angular distribution of deep inelastic collision and quasifission products for ${ }^{78} \mathrm{Kr}+{ }^{40} \mathrm{Ca}$ reaction.
estimating the life-time of dinuclear system for various initial values of energy and angular momentum;

The objects of the research are the reaction products in heavy ion collisions and their angular, mass and charge distributions.

The subjects of the research are total interaction potential between two nuclei, the formation of the dinuclear system at the capture of the projectile-nucleus by the target-nucleus; the evolution of dinuclear system as the dynamics of two interacting nuclei, its lifetime and yield of the reaction products at decay of dinuclear system.

The methods of research. The research methods are mathematical apparatus of phenomenological potential of strong interaction and macroscopic statistical mechanics, analytical and numerical methods for solving differential equations.

The scientific novelty of the research is as follows:
the subbarrier cross sections for the capture of the projectile nucleus with the target nucleus in the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ and ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reactions were calculated for various energies below the potential barrier;
the reasons for the strong difference in the values of the cross sections for evaporation residues in the reactions ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ and ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ for channels 2 n and 3 n , which are associated with the difference in the interactions potentials and in the fusion barrier of these reactions, were revealed;
new approach is proposed based on the assumption that the dinuclear system decays after a $180^{\circ}$ rotation which are explaining the results of observation of the projectile-like products in the forward hemisphere in the reaction ${ }^{78} \mathrm{Kr}$ (10 $\mathrm{MeV} / \mathrm{nuc}$.) $+{ }^{40} \mathrm{Ca}$;
it was shown that, quasifission products during the decay of the dinuclear system in the ${ }^{48} \mathrm{Ti}+{ }^{208} \mathrm{~Pb}$ reaction, are similar in mass distribution to the fission products of a compound nucleus and responsible for an increase in the width of the mass distribution of fusion-fission products.

Practical results of research consist of the following:
for the first time, the mass and charge distributions were obtained for the nonequilibrium stage of the evolution of the dinuclear system formed during capture before complete fusion;
new calculation method has been developed for an alternative interpretation of experimental data with the emission of a projectile like products in the forward hemisphere, as the yield of quasifission products formed during collisions with a large value of the orbital angular momentum.

The reliability of the research results is substantiated by the use of modern methods of nuclear and theoretical physics and highly effective numerical methods and algorithms; careful check of a consistence of the received theoretical results with experimental data and results of other authors; consistency of conclusions with the main provisions of the nuclear reactions at low energy.

## Scientific and practical significance of the research results

The scientific significance of the research results lies in the elucidation of the mechanism of the fusion of massive nuclei in heavy ion collisions at different energies near the Coulomb barrier. The theoretical method improved in the dissertation makes it possible to assess the contribution of each reaction mechanism to the observed reaction products.

The practical significance of the research results lies in the fact that the results obtained make it possible to investigate the role of shell effects, the ratio of the neutron and proton numbers in the formation of reaction products in the production
of transuranic elements.Also, it interpret the appearance of a projectile nucleus in the forward hemisphere outside the target nucleus after dissipation of a sufficient part of its kinetic energy.

Implementation of the research results. Based on the obtained scientific results on the study of the dynamics of interaction in heavy ions collisions at energy near the Coulomb barrier:
calculated subbarrier cross sections for the capture of an incident projectile nucleus with a target nucleus in the reactions ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ and ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$, for various values of energy below the potential barrier were used by international researchers (references in foreign scientific journals Nuclear Physics A 994, 121662, 2020; Physical Review C 101, 014616, 2020; Physical Review C 101, 064604, 2020; Journal of Physics G: Nuclear and Particle Physics 47, 075106, 2020). The use of scientific results made it possible to calculate and evaluate the values of the subbarrier capture cross section for different reactions;
the obtained reasons for the strong difference in the values of the cross sections for evaporation residues in the reactions ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ and ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ are recognized by international researchers (references in foreign scientific journals Nuclear Physics A 994, 121662, 2020; Physical Review C 101, 014616, 2020; Physical Review C 101, 064604, 2020; Journal of Physics G: Nuclear and Particle Physics 47, 075106, 2020) as a promising new theoretical approach to studying the evolution of the dinuclear system in reactions leading to the same compound nucleus. The use of scientific results made it possible to interpret the role of the entrance channel in the evolution of the dinuclear system;
the proposed method explaining the observation of projectile-like products in the front hemisphere in the reaction ${ }^{78} \mathrm{Kr}(10 \mathrm{MeV} / \mathrm{nuc})+.{ }^{40} \mathrm{Ca}$, was used by foreign researchers (references in foreign scientific journals The European Physical Journal A 55, 29, 2019; Journal of Physics G: Nuclear and Particle Physics 47, 045115, 2020; Journal of Physics G: Nuclear and Particle Physics 47, 075106, 2020). The use of scientific results made it possible to investigate the yield of products similar to a projectile and to estimate the lifetime of the dinuclear system;
results on the determination of quasifission products in the decay of the dinuclear system in the ${ }^{48} \mathrm{Ti}+{ }^{208} \mathrm{~Pb}$ reaction were used by foreign scientists (references in international scientific journals Physical Review C 98, 034601, 2018; Physical Review C 98, 014606, 2018; Physical Letters B 803, 135297, 2020 ) when studying the mass distribution in the other reactions. The use of scientific results has made it possible to distinguish the products of different reaction mechanisms that compete during heavy ion collisions at low projectile energies.

Testing of the research results. The research results were reported and tested at 7 international and local scientific conferences.

Publication of the research results. On the theme of the dissertation, 12 scientific works were published, including 4 scientific papers in international scientific journals recommended by the Supreme Attestation Commission of the Republic of Uzbekistan for publishing basic scientific results of PhD dissertations.

Structure and volume of the dissertation. The dissertation is presented on 102 pages consisting of an introduction, four chapters, a conclusion and a bibliography.

## List of publications:

1. Kayumov B.M., Nasirov A.K. Dynamics of capture mechanism in heavy ion collisions at Coulomb barrier energies // Bulletin of the National University of Uzbekistan. - Tashkent (Uzbekistan), 2013. - Vol. 1 - №2 - pp. 169 - 172 (№8 01.00.00).
2. Nasirov A.K., Kayumov B.M. and Yongseok Oh Peculiarities of quasifission reactions in heavy ion collisions // Nuclear Physics A. - Elsevier (Netherland), 2016. - Vol. 946 - pp. 89 -103 (№4. Journal Citation Reports; IF=1.695).
3. Meenu Thakur, Behera B. R., Ruchi Mahajan, et al., Avazbek Nasirov, Bakhodir Kayumov, Binary fragmentation based studies for the near superheavy compound nucleus ${ }^{256} \mathrm{Rf} / /$ European Physical Journal A. - Springer-SIF (Germany), 2017. - Vol. 53 - pp. 133 (№4. Journal Citation Reports; $\mathrm{IF}=2.176$ ).
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8. Nasirov A.K., Giardina G., Mandaglio G., Kayumov B.M. Entrance channel effect in characteristics of the heavy ion reaction products // 62nd DAE-BRNS Symposium on Nuclear Physics, December 20-24. - Patiala (India), 2017 - pp. 11-12.
9. Kayumov B.M., Nasirov A.K. The role of the $N / Z$ - ratio in colliding nuclei during the fusion of sulfur and lead // "II International Scientific Forum Nuclear Science and technologies", June 24-27. - Almaty (Kazakhstan), 2019 - pp. 3031.
10. Nasirov A.K., Kayumov B.M., Mandaglio G., Giardina G., Kim K., Kim Y., The Hindrance to Complete Fusion of Nuclei Related with the Nucleon Transfer Mechanism // "LXIX International Conference "NUCLEUS-2019" on Nuclear Spectroscopy and Nuclear Structure", July 1-5. - Dubna (Russia), 2019 - pp. 276.
11. Nasirov A.K., Kayumov B.M., Yuldasheva G.A., Giardina G., Mandaglio G., Kim K., Kim Y. Different Views of Complete Fusion in Heavy Ion Collisions: Difficulties in Synthesis of Superheavy Elements // "The 9th International Conference on Modern Problems of Nuclear Physics and Nuclear Technologies", September 24-27. - Tashkent (Uzbekistan), 2019 - pp. 17-19.
12. Kayumov B.M., Nasirov A.K., Giardina G., Mandaglio G., Kim K., Kim Y. Mass and Charge Distribution in Heavy Ions Collision // "The 9th International Conference on Modern Problems of Nuclear Physics and Nuclear Technologies", September 24-27. - Tashkent (Uzbekistan), 2019 - pp. 83-85.

## I. Dynamics of initial stage of heavy ion collisions

## § 1.1. Introduction

The theoretical models proposed for the description of deep inelastic collisions [1; pp. 175-210. 2; pp. 490-516. 3; pp. 915-994. 4; pp. 476] the relative motion of nuclei is treated classically and the frictional forces are introduced to describe the kinetic energy dissipation. As a matter of fact, the typical de Broglie wave length associated with the collisions is of the order of one tenth of fm. On the other hand the spatial size of the nuclear system under consideration is a few fm. Therefore, classical mechanics can be considered as a general framework and the question of defining nuclear collective variables will be left out at this stage. The capture of colliding nuclei is a necessity condition to form a compound nucleus. The single barrier penetration models can be applied to describe the capture cross section for light reaction systems [5; pp. 2427-2435]. The fact that colliding nuclei consist of nucleons and the motion of the latter can be described by quantummechanics causes difficulties to describe the dynamics of the collective motion of nuclei and microscopic behavior of nucleons simultaneously. One of microscopic methods, which is used to calculate capture of colliding nuclei and formation of the dinuclear system, is the time dependent Hartree-Fock (TDHF) method. It was applied to nuclei as to many body system [6; pp. 152-201] and to estimate an averaged value of the transferred nucleons between interacting nuclei. Application of the TDHF method to calculate the fusion cross section in case of the massive nuclei failures, since there is hindrance to complete fusion which is called quasifission [6; pp. 152-201]. Therefore, the TDHF method is not used in solving the problems in the synthesis of superheavy elements. By the way, this method can be used to study the formation and shape evolution of the dinuclear system formed at the capture of the projectile-nucleus by the target-nucleus. The authors of Ref. [7; pp. 607-611] have used TDHF method to explore the time dependence of isospin
equilibration via nucleon transfer between fragments of dinuclear system and its lifetime. There is a method of coupling channel to calculate capture and fusion of the relatively light nuclei when mainly there is no hindrance to complete fusion [8; pp. 281-370].

There the authors have used an empirical coupled-channel model, where the coupled-channel effects are considered by including the barrier distribution as a function of the quadrupole and octupole excitation of the colliding nuclei. This method allows to the authors to calculate fusion cross section of the light system when there is no hindrance to the complete fusion. In case of the reactions with the massive nuclei, this method is not suitable, i.e. it cannot be applied to study quasifission channel of the reactions. Sargsyan et al. developed a quantum diffusion approach [9; pp. 064614-12] to calculate the capture process. In this approach, the quantum master equation is used to explore the reduced density matrix. This densitymatrix approach based on the coupled-channel method includes both dissipation and decoherence effects. The authors have explored the probability of capture as a function of the depth of the potential well of the nucleus-nucleus interaction and its curvature. Non-Markovian effects accompanying the passage through the potential barrier have been taken into consideration. It should be noted that the expression of the capture probability with two parameters has been obtained. The one parameter the value of $\tilde{\lambda}$ is related to the strength of linear coupling in coordinates between collective and internal subsystems and the second parameter $\gamma$ is the internalexcitation width, which is used to take into account the non-Markovian effects. This quantum diffusion approach allowed to the authors to describe the set of the experimental data of capture cross section.

In this work, capture of colliding nuclei and evolution of the dinuclear system, which is formed at capture, are explored by estimation of the time dependence of the coupling term between macroscopic variables and microscopic variables characterizing the single-particle states of nucleons in the interacting nuclei.

By identifying collective and macroscopic variables, the dynamical behaviour of the nuclei during the collision is determined by the collective Hamiltonian

$$
H_{\text {coll }}=T_{\text {coll }}+V_{\text {coll }}+V_{\text {coup }}
$$

where $T_{\text {coll }}$ is the kinetic energy term, $V_{\text {coll }}$ is the potential energy depending on the collective variables and $V_{\text {coup }}$ is coupling term between the collective and the intrinsic degrees of freedom. The coupling term is connected with the nucleon transfer between nuclei and constitutes the origin of the nuclear viscosity (friction), corrections to mass parameter and nuclear interaction potential. According to the dissipation-fluctuation theorem friction induces fluctuations around the mean values of the collective variables. Two consequences follow: quantum mechanics is needed for treating the coupling term (nucleonic, degrees of freedom are involved); classical description is meaningful if and only if the amplitude of the fluctuations is small in comparison with the associated quantity.

A classical description of heavy ion reactions may proceed into three steps:

- to define a set of collective variables. This ensemble might be large enough in order to give a sufficient information about the system. On the other hand, it might be small enough in view of practical dynamical calculations;
- to characterize the inertia and friction tensors corresponding to the macroscopic variables as well as the interaction potential;
- to solve the set of equations of motion;

For the set of macroscopic degrees of freedom $Q^{\mu}$, the classical equations of motion are

$$
\frac{d}{d t} \frac{\partial L}{\partial \dot{Q}^{\mu}}-\frac{\partial L}{\partial Q^{\mu}}=-\frac{\partial R}{\partial \dot{Q}^{\mu}}
$$

with the Lagrangian $L=T-V=\frac{1}{2} B_{\mu \nu} \dot{Q}^{\mu} \dot{Q}^{v}-V\left(Q^{\mu}\right)$ and the Rayleigh function $\frac{1}{2} \gamma_{\mu \nu} \dot{Q}^{\mu} \dot{Q}^{\nu}$. $B_{\mu \nu}$ and $\gamma_{\mu \nu}$ are the inertia and the friction tensors. $V\left(Q^{\mu}\right)$ is assumed to be momentum independent.

Such a dynamical study permits to obtain some information about potential, inertia and friction terms as far as initial conditions are perfectly known in heavy ion collisions. They are namely the macroscopic properties of the isolated nuclei and
their relative velocity. The main characteristics of the exit channel are: associated cross sections, energy loss and angular distributions of the final fragments, neutron and proton exchanges.

The capture of incoming projectile-nucleus by target-nucleus is necessary condition for their complete fusion. It is full momentum transfer process leading to formation of dinuclear system (DNS) consisting of projectile and target nuclei: the relative kinetic energy of nuclei is damped being transformed into excitation energy of nucleons and deformation of nuclei. Due to the strong attractive nuclear forces the nuclei are not destroyed but their states are changed by deformation and intensive nucleon exchange between them. The complete kinetic energy relaxation (capture stage) is a main characteristic of the quasifission $(\mathrm{QF})$ reactions. This means that QF takes place only after the capture of the projectile by the target nucleus. Although, QF has generally been understood to occur on short time scales of $10^{-20} \mathrm{~s}$. Where in fusion-fission reactions occur on longer time scales, from $10^{-20} \mathrm{~s}$ to $10^{-16} \mathrm{~s}$. Measurement of fission times can give a definitive signature of fusion-fission.

## § 1.2. Nucleus-nucleus interaction potential

The relative kinetic energy of colliding nuclei is damped partially in deep inelastic collision and interacting nuclei are separated after nucleon exchange. From the theoretical point of view the presence of the potential well of nucleus-nucleus interaction is a necessary condition for capture.

The nucleus-nucleus potential is calculated as follows

$$
\begin{equation*}
V(R)=V_{C}(R)+V_{N}(R)+V_{\text {rot }}(R) \tag{1.1}
\end{equation*}
$$

where $V_{C}(R), V_{N}(R)$ and $V_{\text {rot }}(R)$ are the Coulomb, nuclear, and rotational potentials, respectively. The nuclear shape is important in the calculation of the Coulomb and nuclear interactions between colliding nuclei with orbital angular momentum at the internuclear distance $R$. Thus, the Coulomb interaction of
deformed nuclei can be calculated according to the following expression taken from [10; pp. 766-769]:

$$
\begin{align*}
V_{C}(R) & =\frac{Z_{1} Z_{2}}{R} e^{2}+\frac{Z_{1} Z_{2}}{R^{3}} e^{2}\left\{\left(\frac{9}{20 \pi}\right)^{1 / 2} \sum_{i=1}^{2} R_{0 i}^{2} \beta_{2}^{(i)} P_{2}\left(\cos \alpha^{\prime}{ }_{i}\right)\right. \\
& \left.+\frac{3}{7 \pi} \sum_{i=1}^{2} R_{0 i}^{2}\left[\beta_{2}^{(i)} P_{2}\left(\cos \alpha^{\prime}{ }_{i}\right)\right]^{2}\right\} \tag{1.2}
\end{align*}
$$

where $\alpha^{\prime}{ }_{1}=\alpha_{1}+\Theta, \alpha^{\prime}{ }_{2}=\pi-\left(\alpha_{2}+\Theta\right), \sin \Theta=|L| /(\mu \dot{R} R) ; Z_{i}, \beta_{2}^{(i)}$, and $\alpha^{\prime}{ }_{i}$ are the atomic number (for each fragment), the quadrupole deformation parameter, and the angle between the line connecting the centers of masses of the nuclei (see Fig. 1.1) and the symmetry axis of the fragment $i(i=1,2)$, respectively. Here, $R_{0 i}=$ $r_{0} A_{i}^{1 / 3}, r_{0}=1.16 \mathrm{fm}, e^{2}=1.44 \mathrm{MeV} \cdot \mathrm{fm}$ and $P_{2}\left(\cos \alpha^{\prime}{ }_{i}\right)$ is the second term of the second type of Legendre polynomial; $\mu=M_{1} M_{2} /\left(M_{1}+M_{2}\right)$ is reduced mass of the colliding system consisting from projectile and target with masses $M_{1}$ and $M_{2}$, respectively.


Fig. 1.1. The coordinate systems and angles which were used for the description of the initial orientations of projectile and target nuclei. The beam direction is opposite to $O Z$.

The rotation energy of the deep-inelastic collision is calculated by expression

$$
\begin{equation*}
V_{\text {rot }}(R)=\hbar^{2} \frac{l(l+1)}{2 \mu R^{2}}, \tag{1.3}
\end{equation*}
$$

where $L=l \hbar$; $\hbar$ is Plank's constant.
The relative kinetic energy of colliding nuclei is damped partially in deep inelastic collision and interacting nuclei are separated after nucleon exchange. From the theoretical point of view the presence of the potential well of nucleus-nucleus interaction is a necessary condition for capture.

The nuclear part of the nucleus-nucleus potential is calculated using the folding procedure between the effective nucleon-nucleon forces $f_{\text {eff }}[\rho(x)]$ suggested by Migdal [11; pp. 430] and the nucleon density of the projectile and target nuclei, $\rho_{1}^{(0)}$ and $\rho_{2}^{(0)}$, respectively:

$$
\begin{align*}
V_{\text {nucl }}(R) & =\int \rho_{1}^{(0)}\left(r-r_{1}\right) f_{\text {eff }}[\rho] \rho_{2}^{(0)}\left(r-r_{2}\right) d^{3} r, \\
f_{\text {eff }}[\rho] & =C_{0}\left(f_{\text {in }}+\left(f_{\text {ex }}-f_{\text {in }}\right) \frac{\rho(0)-\rho(r)}{\rho(0)}\right) . \tag{1.4}
\end{align*}
$$

Here $C_{0}=300 \mathrm{MeV} \mathrm{fm}^{3}, f_{\text {in }}=0.09, f_{\text {ex }}=-2.59$ are the constants of the effective nucleon-nucleon interaction; $\rho=\rho_{1}^{(0)}+\rho_{2}^{(0)}$. The effective values of the constants $f_{\text {in }}$ and $f_{e x}$ were fixed from the description of the interaction of the Fermi system by the Grin function method and, therefore, the effect of the exchange term of the nucleon-nucleon interactions were taken into account.

The spherical coordinate system $O$ with the vector $r$, angles $\theta$ and $\phi$ is placed at the mass center of the target nucleus and the $O z$ axis is directed opposite to the beam. In this coordinate system, the direction of the vector $R$ connecting the mass centers of the interacting nuclei has angles $\Theta$ and $\Phi: r_{1}=R$ and $r_{2}=0$. The coordinate system is chosen in such a way that the planes, in which the symmetry axes of nuclei are located, cross the $O z$ line and form the angle $\Phi$. For head-on collisions $\Theta=0$ and $\Phi=\phi$.

The nucleon distribution functions of interacting nuclei in the integrand (1.4) can be expressed using these variables in the same coordinate system $O$. The shape of the dinuclear system nuclei changes with the evolution of the mass asymmetry degrees of freedom: $\beta_{2}=\beta_{2}(Z, A)$ and $\beta_{3}=\beta_{3}(Z, A)$. In order to calculate the potential energy surface as a function of the charge number, we use the values of $\beta_{2}^{\left(2^{+}\right)}$from [12; pp. 1-96] and the values of $\beta_{3}^{\left(3^{-}\right)}$from [13; pp. 55-104]. In the $O$ system the symmetry axis of the target-nucleus is turned around the $\alpha_{2}$ angle, so its nucleon distribution function is as follows:

$$
\begin{gather*}
\rho_{2}^{(0)}(r)=\rho_{0}\left\{1+\exp \left[\frac{r-\tilde{R}_{2}\left(\beta_{2}^{(2)}, \beta_{3}^{(2)} ; \theta^{\prime}{ }_{2}\right)}{a_{0}}\right]\right\}^{-1}  \tag{1.5}\\
\tilde{R}_{2}\left(\beta_{2}^{(2)}, \beta_{3}^{(2)} ; \theta^{\prime}{ }_{2}\right)=R_{0}^{(2)}\left(1+\beta_{2}^{(2)} Y_{20}\left(\theta^{\prime}{ }_{2}\right)+\beta_{3}^{(2)} Y_{30}\left(\theta^{\prime}{ }_{2}\right)\right)
\end{gather*}
$$

where $\rho_{0}=0.17 \mathrm{fm}^{-3}, a_{0}=0.54 \mathrm{fm}$,

$$
\cos \theta_{2}^{\prime}=\cos \theta \cos \left(\pi-\alpha_{2}\right)+\sin \theta \sin \left(\pi-\alpha_{2}\right) \cos \phi
$$

The mass center of the projectile nucleus is shifted to the end of the vector $R$ and its symmetry axis is turned by the angle $\pi-\alpha_{1}$. According to the transformation formulae of the parallel transfer of vectors the variables of the transferred system $O^{\prime}$ are as follows:

$$
\begin{aligned}
& r^{\prime 2}=r^{2}+R^{2}-2 r R \cos \left(\omega_{12}\right) \\
& \cos \left(\omega_{12}\right)=\cos \theta \cos \Theta+\sin \theta \sin \Theta \cos (\phi-\Phi) \\
& \cos \theta_{1}^{\prime}=\frac{(r \cos \theta-R \cos \Theta)}{r^{\prime}} \\
& \cos \phi_{1}^{\prime}=\left(1+\tan ^{2} \phi_{1}^{\prime}\right)^{-1 / 2} \\
& \tan \phi_{1}^{\prime}=\frac{r \sin \phi \sin \theta-R \sin \Theta \sin \Phi}{r \cos \phi \sin \theta-R \sin \Theta \cos \Phi}
\end{aligned}
$$

In the coordinate system $O^{\prime}$, the deviation of the symmetry axis of projectile nuclei relative to the $O^{\prime} z^{\prime}$ axis is determined by the angle

$$
\cos \theta^{\prime \prime}{ }_{1}=\cos \theta_{1}^{\prime} \cos \left(\pi-\alpha_{1}\right)+\sin \theta_{1}^{\prime} \cos \phi_{1}^{\prime}
$$

Now the nucleon distribution function of the projectile-nucleus looks like this

$$
\begin{gather*}
\rho_{1}^{(0)}\left(r^{\prime}\right)=\rho_{0}\left\{1+\exp \left[\frac{r^{\prime}-\tilde{R}_{1}\left(\beta_{2}^{(1)}, \beta_{3}^{(1)} ; \theta_{1}\right)}{a}\right]\right\}^{-1}  \tag{1.6}\\
\tilde{R}_{1}\left(\beta_{2}^{(1)}, \beta_{3}^{(1)} ; \theta^{\prime}{ }_{1}\right)=R_{0}^{(1)}\left(1+\beta_{2}^{(1)} Y_{20}\left(\theta^{\prime}{ }_{1}\right)+\beta_{3}^{(1)} Y_{30}\left(\theta^{\prime}{ }_{1}\right)\right)
\end{gather*}
$$



Fig. 1.2. The influences of the rotational angular momentum on nucleus-nucleus potential; solid line for $\ell=0$; dashed line for $\ell=40$; dotted line for $\ell=60$; dash-dotted line for $\ell=80$.


Fig. 1.3. The influences of the rotational angular momentum (a) and the mutual orientation of colliding nuclei (b) on nucleon-nucleon potential; (a) for $\alpha_{P}=30^{\circ}$ and $\alpha_{T}=45^{\circ}$ solid line $\ell=40$, dashed line $\ell=80$, dotted line $\ell=120$, dashdotted line $\ell=130$; (b) solid, dashed, dotted lines are obtained for projectiletarget orientations $30^{\circ}-90^{\circ}, 45^{\circ}-90^{\circ}, 60^{\circ}-90^{\circ}$, respectively, where $\ell=30$.

The double folding method allows us to take into account dependence on orientation angles of the axial symmetry axis and to analyze their contribution into capture cross section at different values of the initial beam energy. The role of the initial orbital angular momentum and orientation angles of nuclei in the heavy ion collisions can be seen from the Fig. 1.2 and Fig. 1.3. These figures represents the dependence of the depth of the potential well and the Coulomb barrier as functions of the orbital angular momentum for ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb},{ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ and ${ }^{86} \mathrm{Kr}+{ }^{166} \mathrm{Er}$ reactions. It can be clearly seen that the increase of $\ell$ can lead to the reduction of the potential well. It is clear from Fig. 1.3 (b) that, for this reaction with the deformed nuclei the Coulomb barrier depends on the orientation of the axial symmetry axis of interacting nuclei. The Coulomb barrier increases by increase of the orientation angles of the symmetry axis of projectile $\left(\alpha_{P}\right)$ and target $\left(\alpha_{T}\right)$ nuclei relative to the beam. Our model can be applied to calculate the trajectories of collision at energies under, near and above the Coulomb barrier for a given initial orientation angles $\alpha_{P}$ and $\alpha_{T}$ of nuclei.

## § 1.3 Friction coefficient in heavy-ion collisions

When the nuclear forces begin to act between the colliding nuclei, the velocity of their relative motion can be considered as a small quantity compared to the Fermi velocity. Then the speed of the nucleons is mainly associated with their intrinsic motion. Since the relative (collective) motion is rather slow compared to the intrinsic one, the perturbation of the intrinsic motion produced by changing the coupling to the relative motion $(\boldsymbol{R})$ can be assumed to be small during some small time interval $\Delta t$ of an arbitrarily chosen time $t$. The small parameter in our consideration $\Delta t$ thus characterizes the time interval during which the $\boldsymbol{R}$-dependent mean field of the combined dinuclear system changes so little that we can neglect the effect of this changing on the intrinsic motion. At the same time, the characteristic time $\Delta t$ can not be taken smaller than the relaxation time of the mean field. The situation described above is suitable for applying the linear response theory to a description of dissipative heavy-ion collisions. Expressions for the friction coefficients

$$
\begin{gather*}
\gamma_{R}(R(t))=\sum_{i, i \prime}\left|\frac{\partial V_{i i \prime}(R(t))}{\partial R}\right|^{2} B_{i i \prime}^{(1)}(t),  \tag{1.7}\\
\gamma_{\theta}(R(t))=\frac{1}{R^{2}} \sum_{i, i \prime}\left|\frac{\partial V_{i i \prime}(R(t))}{\partial \theta}\right|^{2} B_{i i \prime}^{(1)}(t) \tag{1.8}
\end{gather*}
$$

were obtained in Ref. [14; pp. 373-380. 15; pp. 185] by estimating the evolution of the coupling term between relative motion of nuclei and nucleon motion inside nuclei; $B_{i i \prime}^{(n)}(t)$ is given by:

$$
\begin{align*}
B_{i k}^{(n)}(t)= & \frac{2}{\hbar} \int_{0}^{t} d t^{\prime}\left(t-t^{\prime}\right)^{n} \exp \left(\frac{t \prime-t}{\tau_{i k}}\right)  \tag{1.9}\\
\times & \sin \left[\omega_{i k}\left(R\left(t^{\prime}\right)\right)\left(t-t^{\prime}\right)\right]\left[\tilde{n}_{k}\left(t^{\prime}\right)-\tilde{n}_{i}\left(t^{\prime}\right)\right] \\
& \hbar \omega_{i k}=\varepsilon_{i}+\Lambda_{i i}-\varepsilon_{k}-\Lambda_{k k} \tag{1.10}
\end{align*}
$$

Here $\tilde{n}_{i}$ is the diagonal matrix element of the density matrix which is calculated according to the model presented in Ref. [14; pp. 373-380. 15; pp. 185]; $\tau_{i k}=\tau_{i} \tau_{k} /\left(\tau_{i}+\tau_{k}\right) ; \tau_{i}$ is the life time of the quasiparticle excitations in the single-
particle state $i$ of the nucleus. It determines the damping of single-particle motion. $\tau_{i}$ is calculated using the results of the quantum liquid theory [16; pp.335] and the effective nucleon-nucleon forces from [11; pp. 430]:

$$
\begin{align*}
\frac{1}{\tau_{i}^{(\alpha)}}=\frac{\sqrt{2} \pi}{32 \hbar \varepsilon_{F_{K}}^{(\alpha)}}\left[\left(f_{K}-g\right)^{2}\right. & \left.+\frac{1}{2}\left(f_{K}+g\right)^{2}\right] \times\left[\left(\pi T_{K}\right)^{2}+\left(\tilde{\varepsilon}_{i}-\lambda_{K}^{(\alpha)}\right)^{2}\right] \\
& \times\left[1+\exp \left(\frac{\lambda_{K}^{(\alpha)}-\tilde{\varepsilon}_{i}}{T_{K}}\right)\right]^{-1} \tag{1.11}
\end{align*}
$$

where

$$
\begin{equation*}
T_{K}(t)=3.46 \sqrt{\frac{E_{K}^{*}(t)}{\left\langle A_{K}(t)\right\rangle}} \tag{1.12}
\end{equation*}
$$

is the effective temperature determined by the amount of intrinsic excitation energy $E_{K}^{*}=E_{K}^{*(Z)}+E_{K}^{*(N)}$ and by the mass number $<A_{K}(t)>\left(\right.$ with $<A_{K}(t) \geq=$ $\left.<Z_{K}(t)>+<N_{K}(t)>\right)$. In addition, $\lambda_{K}^{(\alpha)}(t)$ and $E_{K}^{*(\alpha)}(t)$ are the chemical potential and intrinsic excitation energies for the proton $(\alpha=Z)$ and neutron ( $\alpha=$ $N$ ) subsystems of the nucleus $K(K=1$ (projectile), 2 (target)), respectively. Furthermore, the finite size of the nuclei and the difference between the numbers of neutrons and protons makes it necessary to use the following expressions for the Fermi energies [11; pp. 430]:

$$
\begin{align*}
& \varepsilon_{F_{K}}^{(Z)}=\varepsilon_{F}\left[1-\frac{2}{3}\left(1+2{f^{\prime}}_{K}^{\prime}\right) \frac{\left\langle N_{K}>-<Z_{K}>\right.}{\left\langle A_{K}\right\rangle}\right], \\
& \varepsilon_{F_{K}}^{(N)}=\varepsilon_{F}\left[1+\frac{2}{3}\left(1+2 f_{K}^{\prime}\right) \frac{\left\langle N_{K}>-<Z_{K}>\right.}{\left\langle A_{K}\right\rangle}\right], \tag{1.13}
\end{align*}
$$

where $\varepsilon_{F}=37 \mathrm{MeV}$,

$$
\begin{align*}
& f_{K}=f_{\text {in }}-\frac{2}{\left\langle A_{K}\right\rangle^{1 / 3}}\left(f_{\text {in }}-f_{\text {ex }}\right), \\
& f_{K}^{\prime}=f^{\prime}{ }_{\text {in }}-\frac{2}{\left\langle A_{K}\right\rangle^{1 / 3}}\left(f^{\prime}{ }_{\text {in }}-f^{\prime}{ }_{e x}\right) \tag{1.14}
\end{align*}
$$

and $f_{\text {in }}=0.09, f_{\text {in }}^{\prime}=0.42, f_{e x}=-2.59, f_{e x}^{\prime}=0.54, g=0.7$ are the constants of the effective nucleon-nucleon interaction.


Fig. 1.4. (a) The radial friction coefficient and (b) tangential friction coefficient calculated by Eqs. (1.6) and (1.7), respectively, for the ${ }^{78} \mathrm{Kr}+{ }^{40} \mathrm{Ca}$ reaction at different orientation angles of the axial symmetry axis of the projectile and target nucleus.

The dependence of the radial and tangential friction coefficients on the orientation angle of the axial symmetry axis of the projectile and target nucleus is demonstrated in Figs.1.4 (a) and (b), respectively, for the ${ }^{78} \mathrm{Kr}+{ }^{40} \mathrm{Ca}$ reaction. These results are obtained for the initial value of the orbital angular momentum $L=$ $70 \hbar$.


Fig. 1.5. Friction coefficient for the radial motion calculated for the reaction ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ at collision energy $E_{c . m .}=146.41 \mathrm{MeV}$ and angular momentum $0 \hbar$.

The dependence of the friction coefficient on the collision trajectory is presented in Fig. 1.5. The competition between realizations of the capture and deep inelastic collisions in heavy ion collisions depends on the peculiarities of the nucleus - nucleus potential and behavior of radial friction coefficient which is determined by collision energy and shell structure of interacting nuclei.

## § 1.4. Calculation of relative motion of colliding nuclei

The relative motion of colliding nuclei is calculated by equation [17; pp. 205216. 18; pp. 342-369]:

$$
\begin{align*}
& \mu(R) \frac{d \dot{R}}{d t}+\gamma_{R}(R) \dot{R}(t)=F(R)  \tag{1.15}\\
& F\left(R, \alpha_{1}, \alpha_{2}\right)=-\frac{\partial V\left(R, \alpha_{1}, \alpha_{2}\right)}{\partial R}-\dot{R}^{2} \frac{\partial \mu(R)}{\partial R}  \tag{1.16}\\
& \frac{d L}{d t}=\gamma_{\theta}(R) R(t)\left(\dot{\theta} R(t)-\dot{\theta}_{1} R_{1 e f f}-\dot{\theta}_{2} R_{2 e f f}\right)  \tag{1.17}\\
& L_{0}=J_{R}\left(R, \alpha_{1}, \alpha_{2}\right) \dot{\theta}+J_{1} \dot{\theta}_{1}+J_{2} \dot{\theta}_{2} \tag{1.18}
\end{align*}
$$

$$
\begin{equation*}
E_{r o t}=\frac{J_{R}\left(R, \alpha_{1}, \alpha_{2}\right) \dot{\theta}^{2}}{2}+\frac{J_{1} \dot{\theta}_{1}^{2}}{2}+\frac{J_{2} \dot{\theta}_{2}^{2}}{2}, \tag{1.19}
\end{equation*}
$$

where $R \equiv R(t)$ is the relative motion coordinate; $\dot{R}(t)$ is the corresponding velocity; $\alpha_{1}$ and $\alpha_{2}$ are the orientation angles between beam direction and axial symmetry axis of the projectile and target, respectively; $L_{0}$ and $E_{r o t}$ are defined by initial conditions; $J_{R}$ and $\dot{\theta}, J_{1}$ and $\dot{\theta}_{1}, J_{2}$ and $\dot{\theta}_{2}$ are moment of inertia and angular velocities of the DNS and its fragments, respectively; $\gamma_{R}$ and $\gamma_{\theta}$ are the friction coefficients for the relative motion along $R$ and the tangential motion when two nuclei roll on each other's surfaces, respectively; $V\left(R, \alpha_{1}, \alpha_{2}\right)$ is the nucleus-nucleus potential (1.1) calculated by the double folding procedure (1.4).

The moment of inertia of a DNS is calculated by the rigid-body approximation

$$
\begin{equation*}
J_{R}\left(\alpha_{1}, \alpha_{2}, R\right)=\mu R^{2}\left(\alpha_{1}, \alpha_{2}\right)+J_{1}+J_{2} \tag{1.20}
\end{equation*}
$$

where $R^{2}\left(\alpha_{1}, \alpha_{2}\right)$ is the distance between the centers of nuclei corresponding to the bottom of the potential well in the nucleus-nucleus interaction at their given mutual orientations. The moment of inertia of the axial deformed nucleus for the rotation around the axis perpendicular to its axial symmetry is calculated by the expression

$$
\begin{equation*}
J_{i}=\frac{M_{i}}{5}\left(R_{i(\perp)}^{2}+R_{i(\|)}^{2}\right),(i=1,2) \tag{1.21}
\end{equation*}
$$

where $M_{i}$ is the mass of the nucleus; $R_{\perp}\left(\beta_{2}\right)$ and $R_{\|}\left(\beta_{2}\right)$ are the nucleus axes which are perpendicular and parallel to the symmetry axis, respectively:

$$
\begin{aligned}
& R_{\perp}\left(\beta_{2}\right)=R_{0}\left[1+\beta_{2} Y_{20}\left(\frac{\pi}{2}\right)\right] \\
& R_{\|}\left(\beta_{2}\right)=R_{0}\left[1+\beta_{2} Y_{20}(0)\right]
\end{aligned}
$$

Here $R_{0}$ is the spherical equivalent radius.

The difference between capture and deep-inelastic collision is, does the full momentum transfer take place or does not. In both cases the DNS will be formed. Fig. 1.6 illustrates differences of relative motions between capture and deep-inelastic collision. The lifetime of the DNS formed in the capture process will be sufficiently longer in comparison with one of the DNS formed in deep-inelastic collision.


Fig. 1.6. Illustration of capture (a) and deep inelastic collision (b) at heavy ion collisions. Total kinetic energy (TKE) of the relative motion and the part of nucleus-nucleus potential are shown by solid and dotted curves, respectively.

Two conditions must be satisfied for capture: 1) the initial energy $E_{\text {c.m. }}$ of projectile in the center-of-mass system should be sufficiently large to overcome the interaction barrier (Coulomb barrier + rotational energy of the entrance channel), 2) some part of the relative kinetic energy has to be dissipated in order that DNS would be trapped in the well of the nucleus-nucleus interaction potential [17; pp. 205-216.

18; pp. 342-369]. If there is not a potential well the deep-inelastic collision takes place only.

The necessary condition for the occurrence of deep-inelastic collision is dissipation of the initial relative motion due to some mechanisms of nuclear exciting: Coulomb excitation, particle-hole excitation, nucleon transfer and shape deformation. The initial collision energy $E_{c . m}$. in the centre-of-mass system is shared by the kinetic energy $E_{\text {kin }}$ of the relative motion, nucleus-nucleus interaction $V(Z, \ell, R)$ and the dissipated energy $E_{\text {diss }}$ due to the radial and tangential friction forces, which leads to $E_{c . m .}=E_{\text {kin }}+V(Z, \ell, R)+E_{d i s s}$.


Fig. 1.7. (Color online) Results of the dynamical calculations of the total energy $E_{t o t}$ (dot-dashed curves) and the nucleus-nucleus interaction $V(R)$ (solid curves for the incoming path and dashed curves for the outgoing path) as functions of the relative distance $R$ between the centres-of-mass of colliding nuclei in the reaction of ${ }^{78} \mathrm{Kr}+{ }^{40} \mathrm{Ca}$. The presented results are obtained by the use of values $\alpha_{1}=45^{\circ}$ and $\alpha_{2}=15^{\circ}$ of the orientation angles of the axial symmetry of the nuclei relative to the beam direction. The arrows show the points corresponding to the collision

$$
\text { energy } E_{\text {c. } . m}=264 \mathrm{MeV}
$$

In Fig. 1.7 we present the results of the dynamical calculations of the nucleusnucleus interaction $V(Z, \ell, R)$ and the total energy $E_{\text {tot }}=E_{\text {kin }}+V(Z, \ell, R)$ of the relative motion which decreases due to dissipation, which show the difference
between deep-inelastic collisions [Fig. 1.7 (a) and (b)] and the capture process with a full momentum transfer [Fig. 1.7 (c)] in the ${ }^{78} \mathrm{Kr}+{ }^{40} \mathrm{Ca}$ reaction. The solid curves in Fig. 1.7 show the values of $V(R)$ for the incoming path of collisions and the dashed curves are obtained for the outgoing path as functions of the relative distance $R$ between the centres-of-mass of the colliding nuclei. The graphs in Figs. 1.7 (a) and (b) are examples of deep inelastic collisions with the dissipation of the kinetic


Fig. 1.8. Deep inelastic collision (a) and capture (b), (c), (d) trajectories at collision energy $E_{c . m .}=146.41 \mathrm{MeV}$ for the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction. The thick solid lines are the interaction potential $V(t)$; thin solid and dot-dashed lines are the total energy $E_{t o t}$ of the relative motion for the incoming and outgoing trajectories (only for the case of $L=50 \hbar$ ), respectively; the dashed lines are the time-dependent interaction potential $V(t)$ calculated taking into account damping angular momentum and nucleon exchange between nuclei.
energy of relative motion, while the graph in Fig. 1.7 (c) is one of the capture events when the system is trapped into the potential well. These results show that the capture process does not take place in collisions with a large value of the relative angular momentum, for example, at $L=100 \hbar$, when there is no potential well as illustrated in Fig. 1.7 (a). But the collision can be referred to as a deep-inelastic collision in the case of the presence of the potential well if the dissipation of the relative kinetic energy cannot trap the system into the well as shown in Fig. 1.7 (b). The collisions with $L \leq 70 \hbar$ lead to capture processes as the total energy of DNS is trapped into potential well as in Fig. 1.7 (c).


Fig. 1.9. Deep inelastic collision (a), (b) and capture (c) trajectories at collision energy $E_{\text {c.m. }}=143.54 \mathrm{MeV}$ for the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction.

The trajectories of motion for the reaction ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ at collision energies around the Coulomb barrier was calculated by solving the equation of motion (8) for the relative distance and velocity. The bifurcation of the collision trajectories on the deep-inelastic collision and capture as a function of orbital angular momentum ( $L$ ) at the given collision energy ( $E_{c . m}$. in the mass-center system) is calculated by using of the friction coefficient which is determined by the particle-hole excitation of nucleons in nuclei and nucleon exchange between them (1.7), (1.8). Fig. 1.8 shows the dependence of the total energy of radial motion $E_{t o t}$ and nucleus-nucleus potential on the distance $R$ between centers of nuclei for the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction
which determines the capture takes place or not, is demonstrated as a function of $R$ and orbital momentum $L$ for the collision energy $E_{c . m}=146.41 \mathrm{MeV}$. The total energy $E_{\text {tot }}$ decreases due to dissipation by the radial friction coefficient and the dynamical interaction potential $V(t)$ presented by the dotted line while the nucleusnucleus potential including the DNS rotational energy calculated with the undamped values of $L$ is shown by the thick solid line. It is seen that the trajectories with orbital angular momentum $L=0-40 \hbar$ lead to capture because the relative kinetic is enough to overcome barrier increased by rotational energy. The rotational energy is increased by rising $L$ and the system can not overcome potential well, therefore, starting from $L=50 \hbar$ we observe the deep inelastic collisions only.

## § 1.5. Conclusion for Chapter I

In this chapter, the nucleus-nucleus interaction potential, reduced mass and friction coefficient are studied to describe the relative motion of colliding nuclei. The nucleus-nucleus potential consists of the Coulomb and nuclear parts. The Coulomb potential is calculated by the Wong formula [10; pp. 766-769] which allows us to consider the collision of the deformed nuclei. By using double folding method, the interaction potential was calculated for the reactions ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ and ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$. The difference of only two neutrons in the nuclei of the projectile and the target gives us a deeper potential well and a lower barrier for the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction (can be seen in Fig. 1.2). As a result, in the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction, more cases of capture can be observed in comparison with the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction.

The friction forces are related with nucleon exchange between nuclei and their particle-hole excitations. Therefore, friction coefficient is calculated microscopically. The equations of the relative motion was derived for studying the dynamics of colliding nuclei. By using this equation, the trajectories for capture and deep inelastic collision at various initial energies and angular orbital momentums were calculated. The difference between deep-inelastic and capture reactions are connected with the lifetime of the dinuclear system which is formed at heavy ion
collisions at low energies. In the first case the system is not trapped into potential well of nucleus-nucleus interaction and, therefore, the duration of deep-inelastic collisions is shorter than the one of capture reactions when the full momentum of relative motion is transferred into intrinsic degrees freedom and the system is trapped into potential well.

From the results of the energy dissipation of the projectile nucleus (Figs. 1.8 and 1.9), it is clearly seen that for the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction has a capture only in values of the orbital angular momentum below $50 \hbar$. However, for the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction with at the same initial energy, the capture process can be obtained only for a zero value of orbital angular momentum. This was explained by the difference in the height of the barriers in the interaction potentials for both reactions.

Represented results of calculation were published as scientific papers in:

- Kayumov B.M., Nasirov A.K. Dynamics of capture mechanism in heavy ion collisions at Coulomb barrier energies // Bulletin of the National University of Uzbekistan. - Tashkent (Uzbekistan), 2013. - Vol. 1 - №2 - pp. 169-172.
- Nasirov A.K., Kayumov B.M. and Yongseok Oh Peculiarities of quasifission reactions in heavy ion collisions // Nuclear Physics A. - Elsevier (Netherland), 2016. - Vol. 946 - pp. $89-103$.
- Nasirov A.K., Kayumov B.M., Mandaglio G., Giardina G., Kim K. and Kim Y. The effect of the neutron and proton numbers ratio in colliding nuclei on the formation of the evaporation residues in the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ and ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reactions // European Physical Journal A. - Springer-SIF (Germany), 2019. - Vol. 55 - pp. 29.


## II. The charge and mass distributions of reaction

## products

## § 2.1. Introduction

The interaction of nuclei in heavy ion collisions at the around Coulomb barrier energies is multinucleon transfer process. The intensity of the nucleon exchange between nuclei during contact time depends on the energy and impact parameter (orbital angular momentum) of the collision, as well as on their intrinsic structure. The study of the multinucleon transfer (MNT) reaction has called renewed attention to produce new superheavy elements and neutron-rich heavy nuclei. There are different methods of calculation of mass and charge distributions in the reaction products which were developed in 80-th of the last century [2; pp. 319-348, pp. 490516. 19; pp.49-126. 20; pp.115-712.] and their improved versions of nowadays.

The study of the multinucleon transfer (MNT) reaction has called renewed attention to produce new superheavy elements and neutron-rich heavy nuclei [26; pp.278-283].

The calculation of mass and charge distributions in the reaction products were performed by the methods developed in 80-th of the last century [2; pp. 490-516. 19; pp. 49-126. 20; pp. 115-712. 21; pp. 337-676] and by the modern methods [22; pp. 024604-5. 23; pp. 064611-5. 24; pp. 024615-7].

In [22; pp. 024604-5], the yield of products in the multinucleon transfer reaction at the collision energy $E_{\text {c.m. }}$ is calculated the capture cross section

$$
\sigma_{Z, N}\left(E_{c . m .}\right)=\sum_{L} \sigma_{Z, N}\left(E_{\text {c.m. }}, L\right)=\sum_{L} \sigma_{\text {cap }}\left(E_{c . m .,} L\right) Y_{Z, N}\left(E_{c . m .,} L\right)
$$

where $\sigma_{\text {cap }}\left(E_{\text {c.m. }} L\right)$ is partial capture cross section and $Y_{Z, N}\left(E_{c . m .}, L\right)$ is decay probability of the dinuclear system having angular momentum $J$ and with the charge
and neutron numbers $Z$ and $N$ in one of its fragments. The probability of capture $T\left(E_{c . m}, L\right)$ in $\sigma_{\text {cap }}\left(E_{c . m .}, L\right)$

$$
\sigma_{c a p}\left(E_{c . m .} L\right)=\frac{\pi \hbar^{2}}{2 \mu E_{c . m .}}(2 L+1) \cdot T\left(E_{c . m .}, L\right)
$$

is found from the Hill-Wheeler formula:

$$
T\left(E_{\text {c.m. }} L\right)=\left\{1+\exp \left[\frac{2 \pi\left(V_{L}\left(R_{b}, Z, N\right)-E_{c . m .}\right)}{\hbar \omega_{L}(Z, N)}\right]\right\}^{-1}
$$

where $V_{L}\left(R_{b}, Z, N\right)$ is the barrier height of the nucleus-nucleus potential including the Coulomb, nuclear and centrifugal interactions and $R_{b}$ is its position on the internuclear distance $R$; $\omega_{L}$ is a frequency which was used as a parameter in this method. The decay probability $Y_{Z, N}\left(E_{c . m .}, L\right)$ is calculated as the numerical integration of the contribution of the decays with different lifetime of the DNS:

$$
Y_{Z, N}=\Lambda_{Z, N}^{q f} \int_{0}^{t_{0}} P_{Z, N}(t) d t
$$

where $\Lambda_{Z, N}^{q f}$ is the probability of decay of DNS; $P_{Z, N}$ is the charge and neutron distribution of a fragment of DNS. It is calculated by the solutions of the master transport equation. The source of this method is the DNS approach developed in $90^{\text {th }}$ of last century in cooperation of the international group consisted from physicists of Joint Institute for Nuclear Research (Dubna, Russia) and Institute of Nuclear Physics of Academy of Science of Uzbekistan [25; pp. 583-611]. The advance of this method is possibility to include the nuclear structure data in calculations of the nucleon transition coefficients between colliding nuclei. Therefore, many Chinese physicists are used widely it and they have improved the ways of calculations the transition coefficients [26; pp. 278-283. 27; pp. 014618-8. 28; pp. 067601-4].

Dynamics of complete fusion and role of the entrance channel in formation of the reaction products in heavy ion collisions are questionable or they have different
interpretation still nowadays. In the experimental data, the fusion can be strongly hindered by the competing quasifission process, where two touching nuclei reseparate before reaching equilibrium. The mixing the fusion-fission and quasifission contributions due to overlap their mass-angle distributions in measured data leads to ambiguities at estimation of the fusion probability. The significance of this mixing depends on the total mass of colliding nuclei and their mass-asymmetry. If mass and charge numbers of the light nucleus is much smaller than ones of heavy nucleus ( $A_{1} \ll A_{2}$ ) the colliding system is very mass asymmetric. In this case overlap of the mass distributions of the fusion-fission and quasifission is very small since during evolution of dinuclear system, which is formed after capture of projectile by target-nucleus, the nucleon transfer from light fragment to the heavy one is hindered by the barrier at the Businaro-Gallone point of the driving potential. But the mass asymmetry reactions used to synthesize superheavy elements is between the Businaro-Gallone point and mass symmetric region $\left(A_{1} \approx A_{2}\right)$. In this area, the gradient of the potential energy surface produces the forces causing diffusion of nucleons from heavy fragment to light fragment. As a result, the part of mass distribution of quasifission fragments in the mass asymmetric region increases. Therefore, it is actual to study the mass distribution of the fission-like products to find ways to separate quasifission from the fusion-fission products.

## § 2.2. Potential energy surface and driving potential

The dynamics of heavy ion collisions at low energies is determined by the peculiarities of the nucleus-nucleus interaction and shell structure of the interacting nuclei. The landscape of potential energy surface (PES) $U$ plays a main role in an estimation of the complete fusion probability in competition with quasifission. It is calculated as a sum of the reaction energy balance $\left(Q_{g g}\right)$ and the nucleus-nucleus potential $(V(R))$ between interacting nuclei:

$$
\begin{equation*}
U\left(Z, A, \ell, R, \alpha_{1}, \beta_{1}, \alpha_{2}, \beta_{2}\right)=Q_{g g}+V\left(Z, A, R, \ell, \alpha_{1}, \beta_{1}, \alpha_{2}, \beta_{2}\right) \tag{2.1}
\end{equation*}
$$

where $Z=Z_{1}$ and $A=A_{1}$ are charge and mass numbers of a DNS fragment while the ones of another fragment are $Z_{2}=Z_{\text {tot }}-Z_{1}$ and $A_{2}=A_{\text {tot }}-A_{1}$, where $Z_{\text {tot }}$ and $A_{t o t}$ are the total charge and mass numbers of a reaction, respectively; $\beta_{i}$ are the fragment deformation parameters and $\alpha_{i}$ are the orientations relative to the beam direction; $Q_{g g}$ is the reaction energy balance used to determine the excitation energy of $\mathrm{CN}: Q_{g g}=B_{1}+B_{2}-B_{C N}$. The binding energies the initial projectile and target nuclei ( $B_{1}$ and $B_{2}$ ) are obtained from the mass tables in Ref. [29; pp. 337-676], while the one of $\mathrm{CN}\left(B_{C N}\right)$ are obtained from the mass tables [30; pp. 185-381. 31; pp. 1015-1019]. The use of nuclear binding energies including shell effects in the PES and driving potential of DNS leads to the appearance of hollows on the PES around the charge and mass symmetries corresponding to the constituents of DNS with the magic proton or/and neutron numbers.

In Fig. 2.1, the capture stage path is shown by arrow (a) and complete fusion by multinucleon transfer occurs (b) if system overcomes intrinsic fusion barrier. Arrow (c) shows one of possibilities of the DNS quasifission from its more charge symmetric configurations.


Fig. 2.1. Potential energy surface calculated for the DNS leading to formation of the ${ }^{284} 114 \mathrm{CN}$ as a function of the relative distance between the centers of mass of interacting nuclei and mass number of a fragment. The capture stage path is shown by arrow (a) and complete fusion by multinucleon transfer occurs (b) if system overcomes intrinsic fusion barrier. Arrows (c, d) show possibilities of the DNS quasifission.

The driving potential $U_{d r}\left(Z, A, \ell, R_{m}, \alpha_{1}, \beta_{1}, \alpha_{2}, \beta_{2}\right) \equiv U_{d r}(Z, A, \ell)$ is determined by the minimum values of the potential wells for each charge value $Z$. The position of the minimum value of interaction potential on the relative distance is denoted as $R_{m}$. The values of $U_{d r}(Z, A, \ell)$ as a function of angular momentum $\ell$ are found from the data of PES calculated by formula

$$
\begin{equation*}
U_{d r}(Z, A, \ell)=Q_{g g}+V\left(Z, A, \ell, R_{m}, \alpha_{1}, \beta_{1}, \alpha_{2}, \beta_{2}\right) . \tag{2.2}
\end{equation*}
$$

If there is no potential well of $V\left(Z, A, R, \ell, \alpha_{1}, \beta_{1}, \alpha_{2}, \beta_{2}\right)$ at large values of angular momentum or for symmetric massive nuclei, we use $R_{m}$ corresponding to the smallest value of the derivation $\partial V\left(Z, A, R_{m}, \ell, \alpha_{1}, \beta_{1}, \alpha_{2}, \beta_{2}\right) / \partial R$ in the contact area of nuclei.

Fragment mass number ( $A$ )


Fig. 2.2. Driving potential for the reactions which is leading to ${ }^{242} \mathrm{Cf}$ as a function of the fragment charge number.

It can be seen from Fig. 2.2 that the driving potential increases abruptly for the fragment with charge number lower than $\mathrm{Z}=14$ for ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction. The value of the driving potential corresponding to the entrance channel is very low with respect to its maximum value in the fusion direction $Z \rightarrow 0$, the intrinsic fusion barrier, $B_{f u s}^{*}$, becomes larger and the hindrance in completing fusion is very strong. The intrinsic fusion barrier, $B_{f u s}^{*}(Z, A, \ell)$ is determined as the difference between the maximum value of the driving potential between $Z=0$ and $Z=Z_{P}$ and the initial charge value,

$$
\begin{equation*}
B_{f u s}^{*}(Z, A, \ell)=U_{d r}\left(Z_{\max }, A_{\max }, \ell\right)-U_{d r}\left(Z_{P}, A_{P}, \ell\right) \tag{2.3}
\end{equation*}
$$

where $U_{d r}^{\max }=U_{d r}(Z=9) ; Z_{P}=16$ for the reaction with ${ }^{36} \mathrm{~S}$. The value of $B_{f u s}^{*}$ is about 14 MeV for the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction (see Fig. 2.2). The hindrance to the evolution of DNS in direction of the symmetric charge distributions is determined by the barrier $B_{s y m}^{*}$ which is determined in a similar way to the case of $B_{f u s}^{*}$, but the maximum value of the driving potential from symmetric charge region $\left(U_{d r}\left(Z_{\max }^{\text {sym }}, A_{\max }^{\text {sym }}, \ell\right)\right)$ is used.

## § 2.3. Charge distributions of deep-inelastic collision and quasifission products

In deep-inelastic collision and capture, form of DNS is a molecular like system of two interacting nuclei. Nucleons of these nuclei are exchanged, demonstrating an evolution of DNS as a function of proton and neutron numbers. Charge distribution between fragments of DNS and yield of reaction products are calculated by solving the master equation [32; pp. 391-405]. Nucleon exchange leads to release of intrinsic binding energy of nuclei and, therefore, the DNS excitation energy is changed.

The excitation energy of DNS formed in collision of heavy-ions with the energy $E_{\text {c. } m \text {. }}$ and its charge-asymmetry configuration $\left(Z, Z_{t o t}-Z\right)$ is calculated by formula:

$$
\begin{equation*}
E_{Z}^{*}=E_{\text {c. } . \mathrm{m} .}-V\left(Z, R_{m}\right)+\Delta Q_{\mathrm{g} g}(Z), \tag{2.4}
\end{equation*}
$$

where $Z_{\text {tot }}=Z_{1}+Z_{2} ; E_{c . m}$. is the collision energy in the center-of-mass system; $V\left(Z, R_{m}\right)$ is the minimum value of the nucleus-nucleus potential well, calculated at $R=R_{m} ; \Delta Q_{g g}(Z)$ is the change of $Q_{g g}$-value by changing of the DNS charge asymmetry. In this case, the yield of decay fragments is calculated by formula:

$$
\begin{equation*}
Y_{Z}\left(E_{Z}^{*}, \ell, t\right)=P_{Z}\left(E_{Z}^{*}, \ell, t\right) \Lambda_{Z}^{q f}, \tag{2.5}
\end{equation*}
$$

$P_{Z}\left(E_{Z}^{*}, \ell, t\right)$ is the probability of population of the configuration $\left(Z, Z_{t o t}-Z\right)$ at $E_{Z}^{*}$ and $\ell$. The evolution of $Y_{Z}$ is calculated by solving the transport master equation:

$$
\begin{align*}
& \frac{\partial}{d t} P_{Z}\left(E_{Z}^{*}, \ell, t\right)=\Delta_{Z+1}^{(-)} P_{Z+1}\left(E_{Z}^{*}, \ell, t\right)+\Delta_{Z-1}^{(+)} P_{Z-1}\left(E_{Z}^{*}, \ell, t\right)- \\
& -\left(\Delta_{Z}^{(-)}+\Delta_{Z}^{(+)}+\Lambda_{Z}^{q f}\right) P_{Z}\left(E_{Z}^{*}, \ell, t\right), \text { for } Z=2,3, \ldots, Z_{t o t}-2 \tag{2.6}
\end{align*}
$$

Here, the transition coefficients of multinucleon transfer are calculated as in Ref.[33; pp. 228-230]

$$
\begin{equation*}
\Delta_{Z}^{( \pm)}=\frac{1}{\Delta t} \sum_{P, T}\left|g_{P T}^{(Z)}\right|^{2} n_{T, P}^{(Z)}(t)\left(1-n_{P, T}^{(Z)}(t)\right) \frac{\sin ^{2}\left(\Delta t\left(\tilde{\varepsilon}_{P_{Z}}-\tilde{\varepsilon}_{T_{Z}}\right) / 2 \hbar\right)}{\left(\tilde{\varepsilon}_{P_{Z}}-\tilde{\varepsilon}_{T_{Z}}\right)^{2} / 4} \tag{2.7}
\end{equation*}
$$

where the matrix elements $g_{P T}$ describe one-nucleon exchange between the nuclei of DNS, and their values are calculated microscopically using the expression obtained in Ref. [34; pp. 366-370]. In (2.7) we use $\Delta t=10^{-22} \mathrm{~s} \ll t_{\text {int }}$. A nonequilibrium distribution of the excitation energy between the fragments was taken into account in calculations of the single-particle occupation numbers $n_{P}$ and $n_{T}$ as it was done in Ref.[35; pp. 203-210]; $\tilde{\varepsilon}_{P_{Z}}$ and $\tilde{\varepsilon}_{T_{Z}}$ are perturbed energies of singleparticle states. In Eq. 2.6, $\Lambda_{Z}^{q f}$ is the Kramers rate for the decay probability of the dinuclear system into two fragments with charge numbers $Z$ and $Z_{\text {tot }}-Z$ (details in Ref. [36; pp. 034601-18]), and it is proportional to $\exp \left(-B_{q f}(Z) /(k T)\right)$ where $B_{q f}(Z)$ is the quasifission barrier (see Fig. 2.3). Eqs. (2.6) with the coefficients (2.7) and initial condition $P_{Z}\left(E^{*}, 0\right)=\delta_{Z, Z_{P}}$ are solved numerically and the primary mass


Fig. 2.3. Total kinetic energy (solid line) and nucleus-nucleus potential (dashed line) showing the capture case in the ${ }^{48} \mathrm{Ca}+{ }^{248} \mathrm{Cm}$ reaction. The quasifission barrier $B_{q f}$ is taken the a depth of the potential well.
and charge distributions are found for a given interaction time $t_{\text {int }}=5 \cdot 10^{-21} \mathrm{~s}$ (see Ref.[37; pp. 327-365]).

The widths of these "decays" leading to quasifission and complete fusion can be presented by the formula of the width of usual fission [38; pp. 325-339. 39; pp. 384]:

$$
\begin{align*}
& \Lambda_{Z}^{q f}\left(B_{q f}, E_{Z}^{*}\right)=K_{r o t} \omega_{m}\left(\sqrt{\gamma^{2} /\left(2 \mu_{q f}\right)^{2}+\omega_{q f}^{2}}-\gamma /\left(2 \mu_{q f}\right)\right) \\
& \left.\times \exp \left(-B_{q f} / T_{Z}\right)\right) /\left(2 \pi \omega_{q f}\right) \tag{2.8}
\end{align*}
$$

Here $T_{Z}$ is the temperature of the dinuclear system consisting of fragments with charge numbers $Z$ and $Z_{C N}-Z$ :

$$
\begin{equation*}
T_{Z}=\sqrt{8 E_{Z}^{*} /\left(A_{P}+A_{T}\right)} . \tag{2.9}
\end{equation*}
$$

The frequencies $\omega_{m}$ and $\omega_{q f}$ are found by the harmonic oscillator approximation to the nucleus-nucleus potential $V(R)$ shape for the given DNS configuration $\left(Z, Z_{\text {tot }}-Z\right)$ on the bottom of its pocket placed at $R_{m}$ and on the top (quasifission barrier) placed at $R_{q f}$ (see Fig. 2), respectively:

$$
\begin{align*}
& \omega_{m}^{2}=\mu_{q f}^{-1}\left|\frac{\partial^{2} V(R)}{\partial R^{2}}\right|_{R=R_{m}}  \tag{2.10}\\
& \omega_{q f}^{2}=\mu_{q f}^{-1}\left|\frac{\partial^{2} V(R)}{\partial R^{2}}\right|_{R=R_{q f}} \tag{2.11}
\end{align*}
$$

The calculated values of $\hbar \omega_{m}$ and $\hbar \omega_{q f}$ were equal to 46.52 MeV and 22.37 MeV , respectively. The used value of the friction coefficient $\gamma$ is equal to $8 \cdot 10^{-22}$ $\mathrm{MeV} \mathrm{fm}{ }^{-2}$ s which was found from our calculations; $\mu_{q f} \approx \mu=A_{1} \cdot A_{2} / A_{C N}$, where $A_{1}$ and $A_{2}$ are the mass numbers of the quasifission fragments.

The collective enhancement factor of the rotational motion $K_{r o t}$ to the level density should be included because the dinuclear system is a good rotator. It is calculated by the well known expression [40; pp. 635-655]:

$$
K_{r o t}\left(E_{Z}^{*}\right)=\left\{\begin{array}{l}
\left(\sigma_{\perp}^{2}-1\right) f\left(E_{Z}^{*}\right)+1, \quad \text { if } \quad \sigma_{\perp}>1  \tag{2.12}\\
1, \quad \text { if } \sigma_{\perp} \leq 1,
\end{array}\right.
$$

where $\sigma_{\perp}=J_{D N S} T / \hbar^{2} ; \quad f(E)=\left(1+\exp \left[\left(E-E_{c r}\right) / d_{c r}\right]\right) ; \quad E_{c r}=120 \tilde{\beta}_{2}^{2} A^{1 / 3}$ $\mathrm{MeV} ; d_{c r}=1400 \tilde{\beta}_{2}^{2} A^{2 / 3} . \tilde{\beta}$ is the effective quadrupole deformation for the DNS. The moment of inertia of DNS $J_{\perp}^{D N S}$ is found by using Steiner's theorem for the rigid-body moments of inertia of the DNS constituents. It is the moment of inertia of DNS for the rotation around the axis being perpendicular to $\vec{R}$ :

$$
J_{D N S}=J_{1}+J_{2}+M_{1} d_{\perp}^{1}+M_{2} d_{\perp}^{2}
$$

where

$$
J_{i}=\frac{M_{i}\left(a_{i}^{2}+c_{i}^{2}\right)}{5} ; \quad(i=1,2)
$$

are the moment of inertia of the dinuclear system fragments; $a_{\mathrm{i}}$ and $c_{\mathrm{i}}$ are their small and large semi-axes, respectively; $d_{\perp}^{(i)}$ is the distance between the center of mass of the fragment $i(i=1,2)$ and the axis corresponding to the largest moment of inertia of the DNS.

## § 2.4. Mass and charge distribution in the ractions of heavy ion collisions

Due to nucleon exchange between DNS nuclei their mass and charge distributions are changed as functions of time. Their evolution are estimated by solving the transport master equation with the transition coefficients calculated microscopically [25; pp. 583-611. 41; pp. 024604-9]. The proton and neutron systems of nuclei have own energy scheme of the single-particle states and the single-particle schemes depend on the mass and charge numbers of nuclei. Consequently, the transition coefficients $\Delta_{K}^{(-)}$and $\Delta_{K}^{(+)}$of the transport master equation (2.13) being sensitive to the energy scheme and occupation numbers of the single-particle states of the interacting nuclei depend on the mass numbers too. The dependence of the transition coefficients $\Delta_{K}^{(-)}$and $\Delta_{K}^{(+)}$on the mass and charge numbers of nuclei leads to the correlation between proton and neutron numbers in them.

The mass and charge distributions among the DNS fragments are calculated by solving the transport master equation:

$$
\begin{align*}
\frac{\partial}{\partial t} P_{K}\left(E_{K}^{*}, \ell, t\right)= & \Delta_{K+1}^{(-)} P_{K+1}\left(E_{K+1}^{*}, \ell, t\right)+\Delta_{K-1}^{(+)} P_{K-1}\left(E_{K-1}^{*}, \ell, t\right)- \\
& \left(\Delta_{K}^{(-)}+\Delta_{K}^{(+)}+\Lambda_{K}^{\mathrm{q} f}\right) P_{K}\left(E_{K}^{*}, \ell, t\right) \tag{2.13}
\end{align*}
$$

for $K=Z$, $N$ (for proton and neutron transfers). Here $A_{1}=A=N+Z$ is the mass number of the light fragment of DNS while $A_{2}=A_{C N}-A$ and $Z_{2}=Z_{C N}-Z$ are
the mass and charge numbers of the heavy fragment of DNS; $P_{K}\left(A, E_{D N S}^{*}(t), \ell, t\right)$ is the probability of population of the configuration $\left(K, K_{\mathrm{CN}}-K\right)$ of the DNS at the given values of $E_{D N S}^{*}(t), \ell$ and interaction time $t$. To make easy writing of the Eq.(2.13) we have used the following designations:

$$
\begin{aligned}
& P_{K}\left(E_{K}^{*}, \ell, t\right)=P_{K}\left(A, E_{K}^{*}, \ell, t\right), \\
& P_{K \pm 1}\left(E_{K}^{*}, \ell, t\right)=P_{K \pm 1}\left(A \pm 1, E_{K}^{*}, \ell, t\right), \\
& \Delta_{K}^{( \pm)}=\Delta_{K}^{( \pm)}(A), \\
& \Delta_{K \pm 1}^{ \pm)}=\Delta_{K \pm 1}^{( \pm)}(A \pm 1), \\
& \Lambda_{K}^{q f}=\Lambda_{K}^{q f}(A), \\
& E_{K}^{*}=E^{*}(K, A, \ell) .
\end{aligned}
$$

Note these quantities and all quantities characterizing the single-particle states $\tilde{\varepsilon}, n_{P, T}^{(K)}$ and matrix elements $g_{P T}^{(K)}$ in Eq.(2.14) depend on the mass numbers $A$ and $A_{2}=A_{C N}-A$ of the light and heavy fragments, respectively. The transition coefficients of multinucleon transfer are calculated as in [33; pp. 228-230]

$$
\begin{align*}
& \Delta_{K}^{( \pm)}(A)=\frac{4}{\Delta t} \sum_{i_{P}, j_{T}}\left|g_{i_{P} j_{T}}^{(K)}(A)\right|^{2} \times n_{j_{T}, i_{P}}^{(K)}(A, t)\left(1-n_{i_{P}, j_{T}}^{(K)}(A, t)\right) \\
& \quad \times \frac{\sin ^{2}\left[\Delta t\left(\tilde{\varepsilon}_{i_{P}}^{(K)}(A)-\tilde{\varepsilon}_{j_{T}}^{(K)}(A)\right) / 2 \hbar\right]}{\left(\tilde{\varepsilon}_{i_{P}}^{(K)}(A)-\tilde{\varepsilon}_{j_{T}}^{(K)}(A)\right)^{2}} \tag{2.14}
\end{align*}
$$

where the matrix elements $g_{i_{P} j_{T}}^{(K)}(A)$ describe one-nucleon exchange between the DNS nuclei " $P$ " and " $T$ " (for the proton exchange $K=Z$ and for the neutron exchange $K=N$ ) and their values are calculated microscopically as in Ref.[34; pp. 366-370]. Due to dependence of the transition coefficients $\Delta_{K}^{(-)}$and $\Delta_{K}^{(+)}$on the mass and charge numbers of nuclei the neutron and proton distributions $P_{Z}$ and $P_{N}$ are correlated since their master equations are solved parallel way but consequently with the time step $\Delta t$. It is clear that the proton and neutron transfers takes place
simultaneously but with the different probabilities. The letters " $P$ " and " $T$ " are used to indicate the single-particle states of nucleons in projectile-like (light) and targetlike (heavy) fragments, respectively, of DNS. In the present work, we follow the scheme of Ref. [34; pp. 366-370] for estimating these values with $\Delta t=10^{-22} \mathrm{~s} \ll$ $t_{D N S}$, where $t_{D N S}$ is the interaction time of the DNS nuclei and according to calculations it has values $t_{D N S}>5 \cdot 10^{-22} \mathrm{~s}$. This way allows us to take into account non-equilibrium distribution of the excitation energy between the fragments by in calculation of the single-particle occupation numbers $n_{i_{P}}^{(K)}$ and $n_{i_{T}}^{(K)}$ following Ref. [35; pp. 203-210]. The excitation of the DNS is calculated by the estimation of the population of the proton and neutron hole states of one fragment under influence of the mean-field of the other fragment. This kind of evolution of the single-particle occupation numbers $n_{i_{P}}^{(K)}$ and $n_{i_{T}}^{(K)}$ is established by solution of the Liouville quantum equation for the occupation numbers with the linearised collision term:

$$
\begin{equation*}
i \hbar \frac{\partial \tilde{n}_{i_{P}}^{(K)}(t)}{\partial t}=\left[H, \tilde{n}_{i_{P}}^{(K)}\right]+\frac{\tilde{n}_{i_{P}}^{(K)}(t)-n_{i_{P}}^{\mathrm{eq}(K)}\left(T_{Z}\right)}{\tau_{i_{P}}^{(K)}} \tag{2.15}
\end{equation*}
$$

where $H$ is the sum of the collective Hamiltonian $H_{\mathrm{rel}}$ of the relative motion of interacting nuclei of DNS, the secondary quantized Hamiltonian $H_{\mathrm{i} n}$ of the intrinsic motion of nucleons in them and the coupling term $V_{\mathrm{i} n t}$ corresponding to the interaction between collective relative motion of nuclei and intrinsic motion of nucleons,

$$
\begin{equation*}
H=H_{\mathrm{rel}}+H_{\mathrm{i} n}+V_{\mathrm{i} n t} \tag{2.16}
\end{equation*}
$$

The last term $V_{i n t}$ is responsible to excitation of the DNS fragments and it leads to evolution of the occupation numbers of nucleons. The use of the linearised collision term in Eq. (2.14) allows us to determine the time dependent occupation numbers evolve to the thermal equilibrium ones $n_{i_{P}}^{\mathrm{eq}(K)}\left(T_{Z}\right) ; \tau_{i_{P}}^{(K)}$ is the relaxation time of the excited single-particle state $i_{P}$ of the light fragment " $P$ " $\left(i_{T}\right.$ for heavy
fragment " $T$ "). The details of calculation can be find in Refs.[14; pp. 373-380. 35; pp. 203-210]. The thermal equilibrium occupation numbers are calculated by the usual expression:

$$
\begin{equation*}
n^{\mathrm{e} q}\left(T_{Z}\right)=\frac{1}{1+\exp \left[\frac{\left.\tilde{\varepsilon}_{P_{K}}-\varepsilon_{F}\right)}{T_{Z}}\right]} \tag{2.17}
\end{equation*}
$$

where $T_{Z}$ is the effective temperature of DNS with the charge asymmetry $Z$ and its value is determined by the excitation energy $E_{Z}^{*}$ of DNS as the Fermi-gas temperature $T=\sqrt{\frac{E_{Z}^{*}}{a}}$ where $a=1 / 12 \mathrm{MeV}^{-1} . E_{Z}^{*}$ is the excitation energy of DNS and it is determined by the initial beam energy and the minimum of the potential energy as

$$
\begin{equation*}
E_{Z}^{*}(A, \ell)=E_{\mathrm{c} . m .}-V\left(Z, A, R_{m},(\ell)\right)+\Delta Q_{\mathrm{g} g}(Z, A), \tag{2.18}
\end{equation*}
$$

where $V\left(Z, A, R_{m},(\ell)\right)$ is the minimum value of the potential well $V(Z, A, R, \ell)$ at $R_{m} ; \Delta Q_{\mathrm{g} g}(Z, A)=B_{1}+B_{2}-B_{P}-B_{T}$ is included to take into account the change of the intrinsic energy of DNS due to nucleon transitions during its evolution along mass and charge asymmetry axes, where $B_{1}, B_{2}, B_{P}$ and $B_{T}$ are binding energies of the initial (" 1 " and " 2 ") and interacting fragments ("P" and "T") at the given time $t$ of interaction. $\tilde{\varepsilon}_{P_{K}}$ and $\tilde{\varepsilon}_{T_{K}}$ are perturbed energies of single-particle states: $\tilde{\varepsilon}_{i}=\varepsilon_{i}+$ $V_{i i}, V_{i i}$ is the diagonal elements of the matrix $V_{i i}$ (see details in $\operatorname{Ref}(\mathrm{s})$. [35; pp. 203210. 25; pp. 583-611]).

The probability of the yield of the quasifission fragment with the mass and charge numbers, $A$ and $Z$, respectively, after interaction time $t_{\text {int }}$ of DNS is estimated by

$$
\begin{equation*}
Y_{A, Z}\left(E_{Z}^{*}(A), \ell, t_{i n t}\right)=\int_{0}^{t_{i n t}} P_{A, Z}\left(E_{Z}^{*}(A), \ell, t\right) \Lambda_{A, Z}^{q f} d t \tag{2.19}
\end{equation*}
$$

where $P_{A, Z}\left(E_{A, Z}^{*}, \ell, t\right)$ is the probability of population of the configuration $(Z, A)$ of the DNS at the given values of the excitation energy $E_{Z}^{*}(A)$, angular momentum $\ell$
and interaction time $t . \Lambda_{Z}^{q f}$ is the Kramer's rate (2.8) for the decay probability of the DNS into two fragments with charge numbers $Z$ and $Z_{\mathrm{CN}}-Z$. The decay probability increases by decreasing the quasifission barrier $B_{q f}$, which is taken as equal to the depth of the potential well $V(Z, A, R, \ell)$ presented in Fig. 2.3.


Fig. 2.4. The dependence of the neutron distribution as a function of the charge number of the light fragment of the dinuclear system formed in the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction.


Fig. 2.5. The same dependence of the neutron distribution as a function of the charge number of the light fragment of the dinuclear system formed in the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction.

Here, we use only mass and charge distributions $P_{A, Z}=P_{Z}(A, t) \times P_{N}(A, t)$ which are used to find most probable values of $N$ corresponding to the charge numbers $Z$ of the DNS fragments. The results of calculation of neutron distribution for the given proton number for the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ and ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reactions are presented in $\operatorname{Fig}(\mathrm{s}) .2 .4$ and 2.5, respectively. The numbers on the contours show probability of the proton and neutron distributions in the projectile-like fragments of the DNS formed at capture.


Fig. 2.6. The mass number in the projectile nucleus as a function of its proton number calculated for ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ (dot-dashed line) and ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ (dashed line) reactions for non-equilibrium initial stage of the DNS evolution. The equilibrium distribution of neutrons between fragments corresponds to the minimum values of the PES as a function of mass numbers one of the DNS fragments (solid line).

It is obvious from Fig. 2.6 that the projectile-like fragments of the DNS formed in the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction are more neutron rich in comparison with the ones of the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction. As a result the fusion probability is larger in the first reaction. Neutron numbers $N$ corresponding to the given charge numbers presented in Fig. 2.6 are found from the analysis of the parallel solutions of the transport-master equations (2.13): the neutron number $N$ corresponding to the maximum value of the neutron distribution function $P_{N}(A, t),(K=N)$ for the given $Z$ is used in calculation of PES. The equilibrium distribution of neutrons between fragments corresponds to the minimum values of the PES as a function of mass numbers one of the DNS fragments (solid line in Fig. 2.6).

So, the fusion probability of the DNS nuclei is determined by the intrinsic fusion barrier $\left(B_{f u s}^{*}\right)$ and quasifission barrier $\left(B_{q f}\right)$ which are functions of the proton
and neutron numbers (see Ref. [18; pp. 342-369]).
This result has been obtained from the neutron distributions in the light fragment of DNS as a function of its charge number at interaction time $t_{i n t}=6$. $10^{-22} \mathrm{~s}$ after capture (see Fig. 1.6). The time preceding to capture from the beginning the dissipation of the relative energy is about $4 \cdot 10^{-22}-6 \cdot 10^{-22} \mathrm{~s}$ as function of the values of $E_{c . m \text {. }}$ and $\ell$.

The results of the charge number $Z$ and corresponding mass number $A$ are used to calculate PES which allows us to calculate the fusion probability $P_{C N}$ as a function of the mass and charge asymmetry of the DNS nuclei. Therefore, the contributions to the complete fusion of different configurations are different and their ratio depends on the time of calculation. It can be seen from Fig. 2.7 that the driving potential (blue dashed line) calculated for the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction increases abruptly for the fragment with charge number $Z=13$. The value of the driving potential corresponding to the entrance channel $Z=16$ is lower than its maximum value at $Z=13$ in the fusion direction $Z \rightarrow 0$. The increase of the hindrance to complete fusion in the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction in comparison with the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction is seen from the comparison of PES in Figs. 2.8 and 2.9 which are calculated as functions of the intercentre distance between nuclei and their charge-mass asymmetry. The difference between the two PES(s) in Figs. 2.8 and 2.9 appears due to the use of the different mass numbers obtained in the solution of the


Fig. 2.7. Driving potential calculated for the compound nuclei ${ }^{242} \mathrm{Cf}$ reaction as a function of the fragment charge and mass number.


Fig. 2.8. Contour map of the PES calculated for the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction with the non-equilibrium distribution of neutrons between the DNS fragments as a function of the radial distance between their mass centres and charge numbers. The star on
the PES shows the initial charge numbers and internuclear distance.

Eqs. (2.13) by the different initial neutron numbers. As it is seen from Fig. 2.8 the potential surface has higher bump corresponding to the intrinsic fusion barrier, $B_{f u s}^{*}$, in the region $Z=13$ and $R=13.5 \mathrm{fm}$. This bump appears as the hindrance in complete fusion in the case of the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction. This bump is significantly higher than the one on the potential energy surface presented in Fig. 2.9 for the ${ }^{36} \mathrm{~S}$ $+{ }^{206} \mathrm{~Pb}$ reaction. The hindrance to the DNS evolution in the direction of the symmetric charge distributions is determined by the barrier $B_{s y m}^{*}$ which is determined in a similar way to the case of $B_{f u s}^{*}$ but the maximum value of the driving potential from symmetric charge region $\left(U_{d r}\left(Z_{\text {max }}^{\text {sym }}, A_{\text {max }}^{\text {sym }}, \ell\right)\right)$ is used.


Fig. 2.9. The same as in Fig. 2.8 but for the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction.

The presence of the mass asymmetric fission products was studied in the experiment performed at the 15UD Pelletron + LINAC accelerator facility at Inter University Accelerator Centre (IUAC), New Delhi [42; pp. 133-142]. From the mass-energy analysis, a sizeable contribution from the asymmetric fission was observed on the edges of symmetric mass distribution. Evidence of asymmetric fission was also clued from the observed correlation between the masses and
emission angles of the fission fragments. To estimate the amount of the contribution of the quasifission products to the asymmetric fission edges according to the request of the Indian colleagues, we have performed the theoretical calculations in the framework of our dinuclear system model discussed in this Section 2.4. According to the theoretical results, the contribution of quasifission in the mass asymmetric fission region is significant. Comparison of the theoretical and experimental results for the yield of binary reaction products is shown in Fig. 2.10. One can see there that the maximum of the mass asymmetric yield is related with the quasifission products and there is overlap between fusion-fission and quasifission products.


Fig. 2.10. The results of the theoretical estimations of the QF (red dashed curve) and FF (blue dashed curve) products for the ${ }^{48} \mathrm{Ti}(E=273.1 \mathrm{MeV})+{ }^{208} \mathrm{~Pb}$ reaction. The solid curve shows the sum of both yields. The solid circle represents the normalized experimental mass yield [42; pp. 133-142].

The contribution of the quasifission products to the yield of ${ }^{78} \mathrm{Kr}(E / A=$ $10 \mathrm{MeV})+{ }^{40} \mathrm{Ca}$ reactions products have been estimated by the numerical solution of Eq. (2.6). It is clearly seen in Figs. 2.11 and 2.12 that in collisions with $L<60 \hbar$ the centroids of the charge and mass distributions of the quasifission products concentrate at around $Z_{L}=18$ and $A_{L}=38$ for the light product and around $Z_{H}=$

38 and $A_{L}=78$ for the heavy product. The mass numbers shown on the top axis of Figs. 2.11 and 2.12 correspond to those of the primary products of the reaction. The shape of the charge distribution is the manifestation of nuclear shell effects related with the closed shells with the neutron numbers $N=20$ and 40 . The shell effects in the theoretical curves of the the charge distribution of primary products survive due to accumulation of the part of the collision energy in the rotational degrees of freedom (about 40 MeV ) and direct dependence of the transition coefficients on the single-particle energies of nucleons in the DNS nuclei. The gaps between energy


Fig. 2.11. Evolution of the charge distribution of the quasifission products as a function of the lifetime of the DNS formed in the ${ }^{78} \mathrm{Kr}+{ }^{40} \mathrm{Ca}$ reaction at the beam energy of $E_{\text {lab }}=10 \mathrm{MeV} / A$. The mass numbers shown on the top axis of the figure correspond to the primary products of the reaction.
levels in light nuclei are larger than those in massive nuclei and this promotes the appearance of the shell effects. The shape of the charge and mass distributions of the quasifission process depends on the orbital angular momentum. In collisions with
$60 \hbar<L<70 \hbar$ the charge and mass distributions extend up to the mass symmetric region by overlapping with those of the fusion-fission products.


Fig. 2.12. The charge (mass) distribution of the quasifission (dot-dashed and dot-dot-dashed curves) and fusion-fission (dashed curve) products calculated for the ${ }^{78} \mathrm{Kr}+{ }^{40} \mathrm{Ca}$ reaction at the beam energy of $E_{l a b}=10 \mathrm{MeV} / A$. The total yield is shown by the solid line. The mass numbers shown on the top axis correspond to that of the primary products of the reaction.

## § 2.5. Conclusion for Chapter II

The mass and charge distributions between DNS fragments playing an important role in estimation of the complete fusion probability in competition with quasifission have been calculated for the ${ }^{48} \mathrm{Ti}+{ }^{208} \mathrm{~Pb},{ }^{78} \mathrm{Kr}+{ }^{40} \mathrm{Ca},{ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ and ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reactions. The neutron and proton exchanges between the interacting nuclei lead to the change in the mass distribution of the system. The characteristics of the mass distribution of the reaction products are defined by the potential energy
surface of nuclear system, shell structure of protons and neutrons in nuclei and excitation energy of the DNS.

It can be seen, that the shape of the charge and mass distributions of the quasifission process depends on the orbital angular momentum. In collisions with $L<60 \hbar$ the average values of the charge and mass distributions for the ${ }^{78} \mathrm{Kr}+{ }^{40} \mathrm{Ca}$ reaction, are rather concentrated near the projectile/target masses and charges at around ( $Z_{L}=16, A_{L}=38$ ) for the lighter product and at around ( $Z_{H}=38, A_{H}=$ 78) for the heavier product (seen in Fig. 2.12). In collisions with $60 \hbar<L<80 \hbar$ the charge and mass distributions extend up to the mass symmetric region overlapping with those of the fusion-fission products.

The difference between the mass and charge distributions at the given time of the DNS evolution depends on the initial $N / Z$ - ratio in colliding nuclei since transition coefficients causing nucleon transfer are different for the isotopes with different neutron numbers of the nucleus with the same charge numbers. The $N / Z$ ratio has been found by solution of the transport master equations for the proton and neutron distributions between fragments of the DNS formed at capture with the different initial neutron numbers $N=18$ and $N=20$ for the reactions with the ${ }^{34} \mathrm{~S}$ and ${ }^{36} \mathrm{~S}$, respectively.

These reasons are related to the largest of the $N / Z$-ratio in the projectile-like fragments in the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction at the initial non-equilibrium stage of the interaction of the DNS fragments. This is seen from the comparison of the shape of the driving potentials and landscape of PES, which are calculated for these two reactions. In the DNS with the neutron-rich projectile-like fragments formed in the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction the intrinsic fusion barrier is lower. The difference in the mass and charge evolutions for the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ and ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reactions leads to the difference in fusion probabilities in these reactions.

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- Nasirov A.K., Kayumov B.M. and Yongseok Oh Peculiarities of quasifission reactions in heavy ion collisions // Nuclear Physics A. - Elsevier (Netherland),

2016.     - Vol. 946 - pp. 89 - 103.

- Meenu Thakur, Behera B. R., Ruchi Mahajan, et al., Avazbek Nasirov, Bakhodir Kayumov, Binary fragmentation based studies for the near superheavy compound nucleus ${ }^{256} \mathrm{Rf} / /$ European Physical Journal A. - Springer-SIF (Germany), 2017. - Vol. 53 - pp. 133.
- Nasirov A.K., Kayumov B.M., Tashkhodjaev R.B., et al., Mass and angular distributions of the reaction products in heavy ion collisions // IOP Conference Series: Journal of Physics. - IOP Publishing (United Kingdom), 2018. - Vol. 1014 -id. 012009 - 12 p.
- Nasirov A.K., Kayumov B.M., Mandaglio G., Giardina G., Kim K. and Kim Y. The effect of the neutron and proton numbers ratio in colliding nuclei on the formation of the evaporation residues in the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ and ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reactions // European Physical Journal A. - Springer-SIF (Germany), 2019. - Vol. 55 pp. 29.


# III. Cross sections of the capture, complete fusion, quasifission and evaporation residue processes in heavy ion collisions 

## § 3.1. Introduction

In the deep-inelastic collisions, the full momentum transfer of the relative motion does not take place and interaction time of the colliding nuclei is relatively shorter than in the case of capture reactions which request the full momentum transfer. The main difference between deep-inelastic collision and capture events, which can be observed in experiment, is a value of the total kinetic energy of the reaction products. The total kinetic energy of the products formed in the capture reaction are fully damped and its value is significantly lower than the initial collision energy $E_{\mathrm{c} . m}$. while the total kinetic energy of the deep-inelastic collision products is not fully damped and its value is close to the $E_{\text {c.m. }}$. One of the popular methods was the surface friction model suggested by Prof. P. Fröbrich [43; pp. 337-400]. This method allows us to calculate the capture cross sections and it gives good results in calculations of fusion cross section in the reactions with relatively not heavy nuclei. The surface friction model overestimates the fusion cross sections of the reactions with heavy nuclei since it does not take into account quasifission process related with the behaviour of dinuclear system formed at capture. The experimental results related with the study complete fusion reactions shows the appearance of the hindrance in formation of a compound nucleus in heavy ion collisions [44; pp. 282701-4].

The interest to the reaction dynamics had increased after failrure in the synthesis of the superheavy elements with the charge number $\mathrm{Z}>109$ in the experiments of complete fusion. To study the role of the entrance channel at synthesis of superheavy elements the reactions with the different projectile nucleus and the same target nucleus were studied [45; pp. 334-380]. The deviation of the
measured fusion cross section from the theoretical results of the popular methods had been considered as a hindrance in the complete fusion. The hindrance increased by the increase of the mass and change number of the projectile. The energy needed to increase the cross section was called the extra-push energy [46; pp. 113-122]. Later after 10 years it was clear that the extra-push model gives incorrect values of additional energy in case of fusion massive nuclei. Since the beam energy predicted by the extra-push model for the synthesis of the superheavy element $\mathrm{Ds}(\mathrm{Z}=110)$ had given much higher additional energy above the Coulomb barrier and no events were obsered at the predicted energies. The later experimentalists decided to use the beam energy which was found by the linear approximation relative to its value used at synthesis of superheavy elements $\mathrm{Z}=108$ and 109 [47; pp. 125-129]. The experiments showed that the excitation function of the evaporation residues were narrow enough to allow for a safe determination of the positions for the crosssections maxima. The reason of the failure of the use of the beam energy much higher than the Coulomb barrier in cold fusion reactions (with the target nucleus of Pb or Bi ) related with the disappear of the capture possibility. The small size of the well of the nucleus-nucleus potential does not allow to projectle to be trapped due to retricted value of the friction coeffiecient causing dissipation of the relative kinetic energy [17; pp. 205-216]. The DNS formed as a result of the capture of the colliding nuclei can evolve to one of states of the heated and rotating compound nucleus (complete fusion) or it breaks down forming two fragments (quasifission) without reaching the saddle point of CN. Authors of Ref. [17] and [36; pp. 034601-18] showed that quasifission events strongly increase in cold fusion reactions by the increase of the charge number of the projectile. The estimated upper limit of the cross-section for the superheavy nucleus ${ }^{293} 118$ to be about $4.6 \times 10^{-3} \mathrm{pb}$. Indeed, the superheavy element 118 has not obtained yet in the cold fusion reactions.

The mass and charge distributions of the deep-inelastic collision and capture events can widely overlap. This overlaps of the mass and charge distributions have been discussed in Ref. [32; pp. 391-405] for the case of ${ }^{48} \mathrm{Ca}+{ }^{208} \mathrm{~Pb}$ reaction. The main conclusion from this short comment is that capture events are presented as the
yield of the projectile- and target-like products with the total kinetic energy significantly lower than the initial collision energies. The total kinetic energy of the products formed in the capture reaction are around their Coulomb barriers in the exit channels since the amount of the kinetic energy of the relative motion above the Coulomb barrier is dissipated, i.e. the full momentum transfer of the relative motion occurs. The difference between the total kinetic energies of the products formed in the deep-inelastic collision and capture events depends on the projectile-nucleus energy, orbital angular momentum of collision, mass and charge numbers of the colliding nuclei. Unfortunately, there is not so many experimental and theoretical studies devoted to the important problem which allows us to draw interesting conclusions about reaction mechanism of the heavy-ion collisions at the energies near the Coulomb barrier. Since the events producing projectile-like and target-like binary products are considered as the ones of the deep-inelastic collision only. Therefore, the separation of the capture events producing projectile-like and targetlike binary products from the deep-inelastic collision events requests detailed analysis of the experimental data and developing corresponded theoretical methods. Nevertheless, there are papers where the authors have studied the properties of the reaction products by the analysis of their total kinetic energies. For example, in Fig. 3 of Ref.[48; pp. 054603-10], the yield of the binary products with the mass numbers in the range $M_{1}=40-56$ of the ${ }^{50} \mathrm{Cr}+{ }^{208} \mathrm{~Pb}$ reaction and having total kinetic energy around 235 MeV are shown as to be belonged to quasielastic and the ones having the total kinetic energy around 160 MeV are marked as the products of the deep-inelastic collisions. All of the products with the mass numbers in the range $M_{1}=57-82$ are indicated as ones of the fast quasifission process. According to our point of view, among the products marked the deep-inelastic collisions there are events of the quasifission having the total kinetic energy approximately in the range $150-170 \mathrm{MeV}$. More detailed analysis should be performed in our future research devoted to this topic. The yield of the projectile- and target-like products of the capture reactions is responsible for the decrease of the events going to the complete
fusion and this mechanism can be considered as hindrance to complete fusion which is not studied by experimentalists.

## § 3.2. Capture cross section

Two conditions must be satisfied for the capture: 1) the initial energy $E_{\text {c.m. }}$ of a projectile in the center-of-mass system should be enough to reach the potential well of the nucleus-nucleus interaction (Coulomb barrier + rotational energy of the entrance channel) by overcoming or tunneling through the barrier along relative distance in the entrance channel (see in Fig. 1.4 (a)); 2) at the same time the value of the relative kinetic energy above the entrance channel barrier should in correspondence with the size of the potential well: in case of the collision of the massive nuclei the size of the potential is small and, if the initial collision energy is very large relative to the the entrance channel barrier, the dissipation of the kinetic energy may be not enough to make its value lower than barrier of potential well, i.e. to cause trapping into potential well. As a result, the capture does not occur and the deep-inelastic collision takes place (as in Fig. 1.4 (b)). If there is no potential well, the deep-inelastic collision takes place only.

Theoretical values of the capture cross sections are calculated with the quantities characterizing the entrance channel by formula

$$
\begin{equation*}
\sigma_{c a p}\left(E_{\text {c. } . m .}\right)=\frac{\lambda^{2}}{4 \pi} \sum_{\ell=0}^{\ell_{d}}(2 \ell+1) \mathcal{P}_{c a p}^{(\ell)}\left(E_{\text {c.m. }}\right) \tag{3.1}
\end{equation*}
$$

where $\mathcal{P}_{\text {cap }}^{(\ell)}\left(E_{\text {c.m. }}\right)$ is the capture probability of the projectile-nucleus by the targetnucleus in collision with energy $E_{\text {c.m. }}$ and orbital angular momentum $L=\hbar \ell ; \mu$ is the reduced mass of colliding nuclei and $\lambda=\hbar / \sqrt{2 \mu E_{\mathrm{c} . m}}$. All partial waves corresponding to the full momentum transfer events are included into the summation in Eq.(3.1). This means that Eq.(3.1) includes the yield of projectile- and target-like products together with fusion-fission, quasifission and evaporation residue products. The DNS formed in the collisions with the given values of $E_{\text {c.m. }}$ and $\ell$ evolves to
complete fusion due to the transfer of all nucleons of the light fragment to the heavy one or it can decay forming binary products with charge and mass numbers in the wide range. According to our view, the projectile- and target-like products having low total kinetic energy are considered as the quasifission products. The dynamical calculation of mass and charge distributions presented in Chapter 2 allows us to find angular momentum distribution of the DNS formed in capture. In some methods of capture calculations, the variation of the maximum value of the orbital angular momentum $\ell_{d}$ or another way is used to reach an agreement with the experimental values of the capture cross section which is found by ignoring the yield of the capture products which have close values to the initial mass and charge numbers of colliding nuclei [49; pp. 281-370].

The partial capture cross-section $\sigma_{\text {cap }}^{(\ell)}\left(E_{\text {c.m }},\left\{\beta_{i}\right\}\right)$ is determined by calculation of the capture probability $\mathcal{P}_{\text {cap }}^{(\ell)}\left(E_{\text {c.m. }},\left\{\beta_{i}\right\}\right)$ of trapping the curve presenting the dependence of total kinetic energy on the time dependent internuclear distance into the potential well of the nucleus-nucleus interaction:

$$
\begin{equation*}
\sigma_{\text {cap }}^{(\ell)}\left(E_{\text {c.m. } .}\left\{\beta_{i}\right\}\right)=\frac{\lambda^{2}}{4 \pi}(2 \ell+1) \mathcal{P}_{\text {cap }}^{(\ell)}\left(E_{\text {c.m. } .}\left\{\beta_{i}\right\}\right) . \tag{3.2}
\end{equation*}
$$

Here $\lambda$ is the de Broglie wavelength of the entrance channel. The capture probability $\mathcal{P}_{\text {cap }}^{(\ell)}\left(E_{\text {c.m. }},\left\{\beta_{i}\right\}\right)$, which is calculated by classical equation of motion, is equal to 1 or 0 for given beam energy and orbital angular momentum. In dependence on the beam energy, $E_{\text {c.m. }}$. there is a $\ell$-window $\left(\ell_{m}<\ell<\ell_{d}\right)$ for capture as a function of orbital angular momentum:

$$
\mathcal{P}_{c a p}^{(\ell)}\left(E_{\mathrm{c} . m . \mathrm{l}},\left\{\beta_{i}\right\}\right)=\left\{\begin{array}{l}
1, \text { if } \ell_{m}<\ell<\ell_{d} \text { and }  \tag{3.3}\\
E_{\mathrm{c} . m .}>V_{B}, \\
0, \text { if } \ell<\ell_{m} \text { or } \ell>\ell_{d} \text { and } \\
E_{\mathrm{c} . m .}>V_{B} \\
\mathcal{P}_{W K B}^{(\ell)}, \text { for all } \ell \text { if } E_{\mathrm{c} . m .} \leq V_{B}
\end{array}\right.
$$

where $\ell_{m}$ and $\ell_{d}$ are the minimum and maximum values of the orbital angular momentum $\ell$ leading to capture at the given collision energy; $V_{B}$ is the barrier of the nucleus-nucleus potential in the entrance channel; $\mathcal{P}_{W K B}^{(\ell)}$ is probability of the barrier penetrability. It has been calculated by the formula which is derived from the WKB approximation (see Eq. 3.3). The absence of capture at $\ell<\ell_{m}$ means that the total energy curve as a function of $E_{\text {c.m. }}$ is not trapped into potential well: dissipation of the initial kinetic energy is not enough to cause the total energy of DNS to be trapped due to the restricted value of the radial friction coefficient. The number of partial waves giving a contribution to the capture is calculated by the solution of $\mathrm{Eq}(\mathrm{s})$ (1.7)(1.11) for the radial and orbital motions simultaneously.

In sub-barrier capture processes, the barrier penetrability formula is derived from the WKB approximation and it is calculated by:

$$
\begin{equation*}
\mathcal{P}_{W K B}^{(\ell)}\left(E_{\text {c.m. }}\left\{\beta_{i}\right\}\right)=\exp \left[-2 \int_{R_{\text {in }}}^{R_{\text {out }}} k\left(R, \ell,\left\{\beta_{i}\right\}\right) d R\right], \tag{3.4}
\end{equation*}
$$

where

$$
\begin{equation*}
k\left(R, \ell,\left\{\beta_{i}\right\}\right)=\sqrt{\frac{2 \mu}{\hbar^{2}}\left(V\left(R, \ell,\left\{\beta_{i}\right\}\right)-E_{\mathrm{c} . m .}\right)} \tag{3.5}
\end{equation*}
$$

$R_{\text {in }}$ and $R_{\text {out }}$ are inner and outer turning points which were estimated by $V(R)=$ $E_{\text {c. } m \text {. }}$.


Fig. 3.1. Deep-inelastic collision (dashed line), sub-barrier capture (dot-dashed line) and nucleus-nucleus potential for ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ (dotted line) and ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ (solid line).

The colliding nuclei in the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ and ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reactions are spherical in their ground states, therefore, possibility of the population of vibrational states at their excitation is considered. As the amplitudes of the surface vibration we use deformation parameters of first excited $2^{+}$and $3^{-}$states of the colliding nuclei. The values of the deformation parameters of first excited $2^{+}$and $3^{-}$states are are taken from $\operatorname{Ref}(\mathrm{s})$. [12; pp. 1-96] ( $\beta_{2}^{+}$) and [13; pp. 55-104] ( $\beta_{3}^{-}$).

The first step at the estimation of the capture cross section is the calculation of the partial capture cross sections for the seven values for each deformation parameters $\beta_{2}$ and $\beta_{3}$ in the corresponding ranges $-\beta_{2}^{+}<\beta_{2}<\beta_{2}^{+}$and $-\beta_{3}^{-}<$ $\beta_{3}<\beta_{3}^{-}$for the vibrational nuclei, i.e. the differences between intermediate values of the deformation parameters used in calculations are $\Delta \beta_{2}^{+}=\beta_{2}^{+} / 3$. and $\Delta \beta_{2}^{+}=$ $\beta_{2}^{+} / 3$., respectively. This procedure is acceptable since the mean lifetime $\tau$ of first excited $2^{+}$and $3^{-}$states (see Table 1) are much larger than interaction time of
colliding nuclei at capture and complete fusion times which do not precede $10^{-19} \mathrm{~s}$. Therefore, deformation parameters $\beta_{2}$ and $\beta_{3}$ can be considered as the frozen values during the capture process.

| Nucleus | ${ }^{34} \mathrm{~S}$ | ${ }^{36} \mathrm{~S}$ | ${ }^{206} \mathrm{~Pb}$ | ${ }^{208} \mathrm{~Pb}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\beta_{2}^{+}[12 ;$ pp. 1-96] | 0.252 | 0.168 | 0.0322 | 0.055 |
| $\tau_{2^{+}}\left(10^{-12}\right.$ s) [12; pp. 1-96] | 0.023 | 0.110 | 0.11 | 0.0012 |
| $\beta_{3}^{-}[13 ;$ pp. 55-104] | 0.330 | - | 0.083 | 0.100 |
| $\tau_{3^{-}\left(10^{-12} \mathrm{~s}\right)[13 ; \text { pp. 55-104] }}$ | 0.130 | - | - | 47 |

Table 3.1. Deformation parameters $\beta$ and mean lifetime $\tau$ of first excited $2^{+}$and $3^{-}$states of the colliding nuclei used in the calculations in this work.

If the nuclei are spherical, the second stage will be an averaging by the expression (3.6) to find an averaged value of the partial capture cross section over surface vibrational state:

$$
\begin{equation*}
\left\langle\sigma_{\text {cap }}^{(\ell)}\left(E_{c . m}\right)\right\rangle=\int_{-\beta_{2+}}^{\beta_{2+}} \int_{-\beta_{3-}}^{\beta_{3-}} \sigma_{\text {cap }}^{(\ell)}\left(E_{\mathrm{c} . m}, \beta_{2}, \beta_{3}\right) \times g\left(\beta_{2}, \beta_{3}\right) d \beta_{2} d \beta_{3} . \tag{3.6}
\end{equation*}
$$

The surface vibrations are regarded as independent harmonic vibrations and the nuclear radius is considered to be distributed as a Gaussian distribution [50; pp. 147-156],

$$
g\left(\beta_{2}, \beta_{3} ; \alpha\right)=\exp \left[-\frac{\left(\sum_{\lambda} \beta_{\lambda} Y_{\lambda 0}^{*}(\alpha)\right)^{2}}{2 \sigma_{\beta}^{2}}\right]\left(2 \pi \sigma_{\beta}^{2}\right)^{-1 / 2}
$$

where $\alpha$ is the direction of the spherical nucleus. For simplicity, we use $\alpha=0$ :

$$
\sigma_{\beta}^{2}=R_{0}^{2} \sum_{\lambda} \frac{2 \lambda+1}{4 \pi} \frac{\hbar}{2 D_{\lambda} \omega_{\lambda}}=\frac{R_{0}^{2}}{4 \pi} \sum_{\lambda} \beta_{\lambda}^{2}
$$

where $\omega_{\lambda}$ is the frequency and $D_{\lambda}$ is the mass parameter of a collective mode.
The deformation parameters of the vibrational states can be considered as
frozen during the capture process since as it is seen from the Table 1 that the mean lifetime of the first excited states $2^{+}$and $3^{-}$is much longer than the time scale of capture and fusion processes. The time scale of the capture and fusion processes is less than $10^{-19} \mathrm{~s}$.

The role of the initial orbital angular momentum $\ell$ in the heavy ion collisions can be seen from the Fig. 1.2. This figure represents the dependence of the depth of the potential well and the Coulomb barrier as functions of the the orbital angular momentum for ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ and ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reactions. It can be clearly seen that the increase of $\ell$ can lead to reduction of the potential well.

The value of the entrance channel barrier (Coulomb barrier at $\ell=0$ ) calculated for the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction is higher than the one obtained for the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction. The low barrier of the entrance channel is favorable to decrease the CN excitation energy since it makes lower threshold value of the beam energy leading to the CN formation.

The total capture cross section is found by summarizing over partial waves:

$$
\begin{equation*}
\sigma_{c a p}\left(E_{\text {c. } . m}\right)=\sum_{\ell=\ell_{m}}^{\ell=\ell_{d}}\left\langle\sigma_{c a p}^{(\ell)}\left(E_{\text {c. } . m} .\right)\right\rangle . \tag{3.7}
\end{equation*}
$$

It should be noted the range of orbital angular momentum values $\ell_{m}<\ell<$ $\ell_{d}$ contributing to capture cross section depends on the collision energy $E_{\text {c.m. }}$. The calculations have shown that the minimum value of angular momentum is zero ( $\ell_{m}=0$ ) for the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ and ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reactions.

## § 3.3. Formation of compound nucleus and complete fusion cross section

During the lifetime of the excited DNS the composite system evolves by exchanging nucleons between the two nuclei constituting the DNS. For each event, during this stage of reaction, the DNS can reach the shape of a deformed mononucleus or it can decay into two fragments (quasifission) before to reach the
complete fusion stage. In the first case the nuclear system has to reach the statistical equilibrate shape of the CN , but the events which correspond to deformed mononucleus without barrier to provide stability $\left(B_{f}=0\right.$ for $\ell>\ell_{f}$, where the $\ell_{f}$ is characteristic of each CN with its structure of nucleons) cannot reach the shape of CN because the deformed complete fusion system immediately decays into two fragments (fast fission process). Therefore, the partial capture cross section is contributed by the following terms:
$\sigma_{\text {cap }}^{(\ell)}\left(E_{\text {c. } m .},\left\{\beta_{i}\right\}\right)=\sigma_{q f}^{(\ell)}\left(E_{\text {c. } m .},\left\{\beta_{i}\right\}\right)+\sigma_{f u s}^{(\ell)}\left(E_{\text {c. } . m .},\left\{\beta_{i}\right\}\right)+\sigma_{f f}^{(\ell)}\left(E_{\text {c. } . m .},\left\{\beta_{i}\right\}\right)$.

The partial fusion cross section is determined by the product of capture cross section $\sigma_{c a p}^{(\ell)}\left(E_{\mathrm{c} . m}, \beta_{2}, \beta_{3}, \alpha_{\mathrm{T}}, \alpha_{\mathrm{P}}\right)$ (for simplicity we use $\sigma_{c a p}^{(\ell)}\left(E_{\mathrm{CN}}^{*},\left\{\beta_{i}\right\}\right)$ ) and the fusion probability $P_{C N}$ of DNS for the various excitation energies [51; pp. 75-87. 52; pp. 064608-14. 53; pp. 064614-14. 54; pp. 1639-1650. 55; pp. 425-430. 56; pp. 2635-2645] by the use of formula:

$$
\begin{equation*}
\sigma_{\text {fus }}^{(\ell)}\left(E_{\mathrm{c} . m .,}\left\{\beta_{i}\right\}\right)=P_{C N}\left(E_{\mathrm{c} . m .}, \ell,\left\{\beta_{i}\right\}\right) \sigma_{\text {cap }}^{(\ell)}\left(E_{\mathrm{c} . m .,}\left\{\beta_{i}\right\}\right), \tag{3.9}
\end{equation*}
$$

The fusion probability $P_{C N}\left(E_{\text {c.m. }}, \ell,\left\{\beta_{i}\right\}\right)$ is calculated as the sum of contributions to complete fusion from the charge symmetric configuration $Z_{\text {sym }}$ of DNS up to configuration corresponding to the maximum value of the driving potential $Z_{\max }$ :

$$
\begin{equation*}
P_{C N}\left(E_{\mathrm{c} . m .}, \ell,\left\{\beta_{i}\right\}\right)=\sum_{Z_{\text {sym }}}^{Z_{\max }} P_{Z}\left(E_{Z}^{*}, \ell\right) P_{C N}^{(Z)}\left(E_{Z}^{*}, \ell,\left\{\beta_{i}\right\}\right) \tag{3.10}
\end{equation*}
$$

where $E_{Z}^{*}$ is calculated by formula (2.4) and the weight function $P_{Z}\left(E_{Z}^{*}, \ell\right)$ is the mass and charge distributions probability $P_{Z}\left(E_{Z}^{*}, \ell, t\right)$ in the DNS fragments is determined by solution of the transport master equation (2.6); the fusion probability $P_{C N}^{(Z)}(A)$ from the charge $(Z)$ and mass $(A)$ asymmetry configuration of the DNS is calculated as the branching ratio $P_{C N}^{(Z)}\left(E_{Z}^{*}, \ell ;\left\{\alpha_{i}\right\}\right)$ of widths related to the
overflowing over the quasifission barrier $B_{q f}(Z)$ at a given mass asymmetry, over the intrinsic barrier $B_{f u s}(Z)$ on mass asymmetry axis to complete fusion and over $B_{\text {sym }}(Z)$ in opposite direction to the symmetric configuration of the DNS:

$$
\begin{equation*}
P_{C N}^{(Z)} \approx \frac{\Gamma_{f u s}(Z)}{\Gamma_{q f i s s}(Z)+\Gamma_{f u s}(Z)+\Gamma_{s y m}(Z)} . \tag{3.11}
\end{equation*}
$$

Here, the complete fusion process is considered as the evolution of the DNS along the mass asymmetry axis overcoming $B_{f u s}(Z)$ (a saddle point between $Z=0$ and $Z=Z_{P}=16$ ) and ending in the region around $Z=0$ or $Z=Z_{C N}$ (fig. 2.10). The evolution of the DNS in the direction of the symmetric configuration increases the number of events leading to quasifission of more symmetric masses. This kind of channels are taken into account by the term $\Gamma_{s y m}(Z)$. One of the similar ways was used in Ref.[57; pp. 044601-5]. The complete fusion can be presented by the formula of the width of usual fission [39; pp. 384]:

$$
\begin{equation*}
\Gamma_{f u s}(Z)=\frac{\rho_{f u s}\left(E_{Z}^{*}\right) T_{Z}}{2 \pi \rho\left(E_{Z}^{*}\right)}\left(1-\exp \frac{\left(B_{f u s}(Z)-E_{Z}^{*}\right)}{T_{Z}}\right) \tag{3.12}
\end{equation*}
$$

where $\rho_{f u s}\left(E_{Z}^{*}\right)=\rho\left(E_{Z}^{*}-B_{\mathrm{fus}}(Z)\right)$; usually the value of the factor $(1-$ $\left.\exp \left[\left(B_{i}(Z)-E_{Z}^{*}\right) / T_{Z}\right]\right)$ in (3.12) is approximately equal to the unit.

Inserting Eq. (3.12) to (3.11), we obtain the expression (3.13) used in our calculations [38; pp. 325-339]:

$$
\begin{equation*}
P_{C N}^{(Z)}\left(E_{Z}^{*}\right)=\frac{\rho_{f u s}\left(E_{Z}^{*}\right)}{\rho_{f u s}\left(E_{Z}^{*}\right)+\rho_{\mathrm{q} f i s s}\left(E_{Z}^{*}\right)+\rho_{\mathrm{s} y m}\left(E_{Z}^{*}\right)} . \tag{3.13}
\end{equation*}
$$

Putting the level density function of the Fermi system leads to formula of the calculation of fusion probability for the given values of the DNS excitation energy $E_{Z}^{*}$ and angular momentum $L$ from its charge asymmetry $Z$ :

$$
\begin{equation*}
P_{C N}^{(Z)}\left(E_{Z}^{*}\right)=\frac{e^{-B_{f u s}^{*(Z)} / T_{Z}}}{e^{-B_{f u s}^{*(Z)} / T_{Z}}+e^{-B_{q f i s s}^{*(Z)} / T_{Z}}+e^{-B_{s y m}^{*(Z)} / T_{Z}}} \tag{3.14}
\end{equation*}
$$

The fusion cross section is calculated by summarizing contributions of all partial waves (angular momentum):

$$
\begin{equation*}
\sigma_{f u s}\left(E_{\mathrm{c} . m .}\right)=\sum_{\ell=0}^{\ell=\ell_{f}}\left\langle\sigma_{f u s}^{(\ell)}\left(E_{\mathrm{c} . m .}\right)\right\rangle \tag{3.15}
\end{equation*}
$$

The averaged value of the partial fusion cross section is calculated by the same method as in the case of partial capture cross section (Eq. 3.6):

$$
\begin{align*}
\sigma_{f u s}^{(\ell)}\left(E_{c . m}, \alpha_{\mathrm{T}}, \alpha_{\mathrm{P}}\right)= & \int_{-\beta_{2+}}^{\beta_{2+}} \int_{-\beta_{3-}}^{\beta_{3-}} \sigma_{f u s}^{(\ell)}\left(E_{\mathrm{c} . m}, \beta_{2}, \beta_{3}, \alpha_{\mathrm{T}}, \alpha_{\mathrm{P}}\right) \times  \tag{3.16}\\
& g\left(\beta_{2}, \beta_{3}\right) d \beta_{2} d \beta_{3} .
\end{align*}
$$

Calculation of the final results for the deformed nuclei with orientation angles ( $\alpha_{P}$ and $\alpha_{T}$ ) relative to the beam direction, will be [52; pp. 064608-14]:

$$
\begin{equation*}
\left\langle\sigma_{f u s}^{(\ell)}\left(E_{c . m}\right)\right\rangle=\int_{0}^{\pi / 2} \sin \alpha_{P} \int_{0}^{\pi / 2} \sin \alpha_{T} \times \sigma_{f u s}^{(\ell)}\left(E_{c . m}, \alpha_{T}, \alpha_{P}\right) d \alpha_{P} d \alpha_{T} \tag{3.17}
\end{equation*}
$$

By taking into account eqs. (3.16) and (3.17), we can rewrite (3.15) as:

$$
\begin{equation*}
\sigma_{f u s}\left(E_{c . m .}\right)=\sum_{\ell=0}^{\ell=\ell_{f}}(2 \ell+1) \cdot \sigma_{c a p}^{(\ell)}\left(E_{c . m .}\right) \cdot P_{C N}^{(\ell)}\left(E_{c . m .}\right) \tag{3.18}
\end{equation*}
$$



Fig. 3.2. Partial fusion cross sections calculated for ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ (red solid curves) and ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reactions (black dashed curves) as a function of the angular momentum, for differnet CN excitation energy.

Fig. 3.2 represents results for partical fusion cross section for different excitation energy. It can be seen, that, by decreasing energy of collision reaction ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ has less probability of fusion comparing to ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction. Also fusion prosesses take places in all values of angular momentum for the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction. For the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction, in the energy region $E_{\mathrm{C} N}^{*}=24,26 \mathrm{MeV}$, contribution of deep inelastic collision and quasifission are increasing due to higher value of the Coulomb barrier comparing to ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction (can be seen in Fig. 1.2).

In Fig. 3.3 the capture and complete fusion cross sections are compared with the experimental data. The excitation energy of the compound nucleus $E_{\mathrm{C} N}^{*}=$ $E_{\mathrm{c} . m .}+Q_{g g}$ corresponding to the collision energy in the center of mass system $E_{\text {c. } m}$.
has been used for the convenience of comparison of the corresponding experimental and theoretical cross sections of these reactions. It is clearly seen in Fig. 3.3 that the excitation fusion function of the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ is much higher than the one of the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction in the energy region $E_{\mathrm{C} N}^{*}=24-35 \mathrm{MeV}$, which corresponds to experimental results [58; pp. 064602-6].


Fig. 3.3. Capture and complete fusion cross sections calculated for ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ (thick and thin dashed curves) and ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reactions (thick and thin solid curves) as a function of the CN excitation energy are compared with the experimental data [58; pp. 064602-6].

The lower threshold energy $E_{\mathrm{CN}}^{* \min }$ of the fusion excitation function is determined by the height of the Coulomb barrier in the entrance channel and reaction balance energy $Q_{g g}$. The large negative values of $Q_{g g}$ decrease the value of $E_{\mathrm{CN}}^{* \min }$ [52; pp. 064608-14]. The $Q_{g g}$-values are equal to -113.79 and -111.02 MeV for the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ and ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reactions, respectively. As it was discussed above as well as according to Fig. 1.2, the Coulomb barrier of the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction is lower than the one of the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction. Consequently, the threshold energy $E_{\mathrm{CN}}^{* m i n}$ for the first reaction is significantly lower than the one for the second reaction.


Fig. 3.4. Fusion probability $P_{\text {CN }}$ calculated for the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ (red solid line) and ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ (blue dashed line) reactions.

Comparison of the fusion probabilities $P_{\mathrm{C} N}$ calculated for the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ and ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reactions is presented in Fig. 3.4. It is seen that the complete fusion probability of the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction is about one and half times larger than that of the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction.

## § 3.4 Quasifission and fast fission of mononucleus processes in evolution of DNS

Another binary process, which leads to the formation of two fragments similar to quasifission, is the fast fission. According to the liquid-drop model, the fast fission occurs only at large values of the angular momentum, $\ell>\ell_{f}$, causing disappearance of the macroscopic fission barrier, $B_{f i s}(\ell)$, of the rotating nucleus [59; pp. 2039-2053]. It is the disintegration of the fast rotating mononucleus, which
survives quasifission (the decay of the DNS into two fragments without formation of CN ), into two fragments.

In the case of very heavy nucleus ( $Z>106$ ), the fission barrier providing their stability against fission, appears only due to shell effects in their binding energy [60; pp. 292-349]. The damping of the shell effects decreases the possibility of the mononucleus to reach the CN equilibrium shape, and the mononucleus breaks down into two fragments without reaching the CN shape. The fission barrier consists of the contributions of the macroscopic and microscopic parts. The dependence of the fission barrier $B_{f i s}$ including shell correction $\delta W$ on the critical angular momentum $\ell_{f}$, and can be determined by the formula:

$$
\begin{equation*}
B_{f i s}(\ell, T)=c B_{f i s}^{m}(\ell)-h(T) \cdot q(\ell) \cdot \delta W \tag{3.19}
\end{equation*}
$$

Here, $B_{f i s}^{m}$ is macroscopic barrier in formation of CN [59; pp. 2039-2053]. The microscopic (shell) correction to the fission barrier $\delta W=\delta W_{s a d}-\delta W_{g s} \sim \delta W_{g s}$ is taken from the table [31; pp. 1015-1019. 61; pp. 1681-1747. 62; pp. 014303-10. 62; pp. 914-918]. The damping of the microscopic fission barrier on the excitation energy and angular momentum of CN is considered by the formulae used in ref. [64; pp. 064607-9],

$$
\begin{equation*}
h(T)=\left\{1+\exp \left[\left(T-T_{0}\right) / d\right]\right\}^{-1} \tag{3.20}
\end{equation*}
$$

and

$$
\begin{equation*}
q(\ell)=\left\{1+\exp \left[\left(\ell-\ell_{1 / 2}\right) / \Delta \ell\right]\right\}^{-1} \tag{3.21}
\end{equation*}
$$

where, in formula (3.20), $T=\sqrt{E^{*} / a}$ represents the nuclear temperature depending on the excitation energy and the level density parameter $a, d=0.3 \mathrm{MeV}$ is the rate of washing out the shell corrections with the temperature and $T_{0}=1.16 \mathrm{MeV}$ is the value at which the damping factor $h(T)$ is reduced by $1 / 2$; in $(3.21), \ell_{1 / 2}=20 \hbar$ is the value at which the damping factor $q(\ell)$ is reduced by $1 / 2$ and $\Delta \ell=3 \hbar$ is the rate of washing out the shell corrections with the angular momentum. In addition,
this procedure allows the shell corrections to become dynamical quantities. Therefore, if the capture process of the beam by the target takes place, the fission barrier disappears at $\ell>40 \hbar$ due to damping of the shell correction by $q(\ell)$.

The partial fusion cross section with $\ell>\ell_{f}$ is considered as a partial fast fission cross section. We should stress that for the superheavy elements $\ell_{f}$ is not relevant quantity because there is no barrier connected with the liquid-drop model. The fast fission cross section is calculated by summing the contributions of the partial waves corresponding to the range $\ell_{f} \leq \ell \leq \ell_{d}$ leading to the formation of the mononucleus,

$$
\begin{equation*}
\sigma_{f f}\left(E_{\mathrm{c} . m .}\right)=\sum_{\ell=\ell_{f}}^{\ell=\ell_{d}}(2 \ell+1) \cdot \sigma_{c a p}^{(\ell)}\left(E_{\mathrm{c} . m .}\right) \cdot P_{C N}^{(\ell)}\left(E_{\mathrm{c} . m .}\right) \tag{3.22}
\end{equation*}
$$

The capture cross section in the framework of the DNS model is equal to the sum of the quasifission, fission, and fast fission cross sections, see Eq. (3.8). It is clear that the fusion cross section includes the cross sections of ERs and fusion fission products. Obviously, the quasifission cross section is defined by

$$
\begin{equation*}
\sigma_{q f}\left(E_{\mathrm{c} . m .}\right)=\sum_{\ell=0}^{\ell=\ell}(2 \ell+1) \cdot \sigma_{c a p}^{(\ell)}\left(E_{\mathrm{c} . m .}\right) \cdot\left[1-P_{C N}^{(\ell)}\left(E_{\mathrm{c} . m .}\right)\right] . \tag{3.23}
\end{equation*}
$$

## § 3.5 Evaporation recidue cross section

In the DNS framework the partial cross sections of the CN formation are used to calculate evaporation residue (ER) cross sections at given values of the CN excitation energy $E_{\mathrm{CN}}^{*}$ and angular momentum $\ell$ by the advanced statistical model [64; pp. 064607-9]

$$
\begin{equation*}
\sigma_{\mathrm{ER}}^{x}\left(E_{x}^{*}\right)=\sum_{\ell=0}^{\ell=\ell}(2 \ell+1) \cdot \sigma_{\mathrm{ER}}^{x}\left(E_{x}^{*}, \ell\right) \tag{3.24}
\end{equation*}
$$

where, $\sigma_{\mathrm{ER}}^{x}\left(E_{x}^{*}, \ell\right)$ is the partial cross section of ER formation obtained after the emission of particles $v(x) n+y(x) p+k(x) \alpha+s(x)$ (where $v(x), y(x), k$, and $s$ are numbers of neutrons, protons, $\alpha$ particles, and $\gamma$ quanta) from the intermediate
nucleus with excitation energy $E_{x}^{*}$ at each step $x$ of the de-excitation cascade by the formula (see Refs. [17; pp. 205-216. 18; pp. 342-369. 64; pp. 064607-9])

$$
\begin{equation*}
\sigma_{\mathrm{ER}}^{x}\left(E_{x}^{*}, \ell\right)=\sigma_{\mathrm{ER}}^{x}\left(E_{x-1}^{*}, \ell\right) \cdot W_{\mathrm{sur}}^{x}\left(E_{x}^{*}, \ell\right) . \tag{3.25}
\end{equation*}
$$

In Eq. (3.25), $\sigma_{\mathrm{ER}}^{x}\left(E_{x-1}^{*}, \ell\right)$ is the partial cross section of the intermediate excited nucleus formation at the $(x-1)$ th step, and $W_{\text {sur }}^{x}\left(E_{x}^{*}, \ell\right)$ is the survival probability of the $x$ th intermediate nucleus against fission along the de-excitation cascade of CN ; obviously

$$
\sigma_{\mathrm{ER}}^{(0)}\left(E_{0}^{*}, \ell\right)=\sigma_{f u s}\left(E_{\mathrm{CN}}^{*}, \ell\right)
$$

i.e., the first evaporation starts from the heated and rotating CN and $E_{0}^{*}=E_{C N}^{*}=$ $E_{\text {c.m. }}+Q_{g g}-V_{\text {rot }}(\ell) ; V_{\text {rot }}(\ell)$ is the rotational energy of the CN .

The fission barrier decreases by the increase of the angular momentum $\ell$ and, therefore, in calculation of $W_{\text {sur }}^{(x-1)}\left(E_{\chi}^{*}, \ell\right)$ the fission barrier is used a sum of the parameterized macroscopic fission barrier $B_{f i s}(\ell)$ depending on the angular momentum and the shell correction $\delta W$, eq. (3.19).

The survival probability under the evaporation of $x$ neutrons is

$$
\begin{equation*}
W_{s u r}^{x}\left(E_{x}^{*}, \ell\right)=P_{x n}\left(E_{x}^{*}, \ell\right) \cdot \prod_{i=1}^{i_{\max }=x}\left[\frac{\Gamma_{n}}{\Gamma_{n}+\Gamma_{f}}\right] \tag{3.26}
\end{equation*}
$$

where the index $i$ is equal to the number of emitted neutrons, $P_{x n}\left(E_{x}^{*}, \ell\right)$ is the probability of emitting exactly $x$ neutrons [65; pp. 767-779], $\Gamma_{n}$ and $\Gamma_{f}$ represent the decay width of neutron evaporation and fission respectively. To calculate $\Gamma_{n} / \Gamma_{f}$, Vandenbosch and Huizenga [66; pp. 223] have suggested a classical formalism:
$\frac{\Gamma_{n}}{\Gamma_{f}}=\frac{4 A^{\frac{2}{3}} a_{f}\left(E_{x}^{*}-B_{n}\right)}{K_{0} a_{n}\left[2 a_{f}^{1 / 2}\left(E_{x}^{*}-B_{f}\right)^{1 / 2}-1\right]} \exp \left[2 a_{n}^{1 / 2}\left(E_{x}^{*}-B_{n}\right)^{1 / 2}-2 a_{f}^{1 / 2}\left(E_{x}^{*}-B_{f}\right)^{1 / 2}\right]$
where $A$ is the mass number of the nucleus considered, $B_{n}$ neutron separation energy. The constant $K_{0}$ is taken as $10 \mathrm{MeV}, a_{n}=A / 10$ and $a_{f}=1.1 \cdot a_{n}$, are the level density parameters of the daughter nucleus and the fissioning nucleus at the ground state and saddle configurations respectively. $B_{f}=B_{f i s}(\ell)$ the fission barrier and this height is a decisive quantity in the competition between processes of neutron evaporation and fission.


Fig. 3.5. The theoretical values of capture (red thick dashed curve), complete fusion (solid thick curve) and ER (thin solid-2n, thin dashed-3n, thin dotted-4n and thin dot-dashed-5n channels) cross sections are compared with the experimental values of the capture (red circles) and ER (black squares-2n and red triangles-3n channels) cross sections of the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ [58; pp. 064602-6].

If the colliding nuclei are deformed, the possibility of collision with arbitrary orientation angles of their symmetry axis should be considered. Due to the dependencies of the nucleus-nucleus potential $(V)$ and moment of inertia for DNS $\left(J_{R}\right)$ on the orientation angles of the axial symmetry axis of the deformed nuclei, the
excitation function of the capture and fusion are sensitive to the values of orientation angles. In the case of spherical nuclei, we can take into account of the vibrational excitation of their surfaces due to interactions.


Fig. 3.6. The same as in Fig. 3.5 but for the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction [58; pp. 064602$6]$.

The theoretical values of capture, complete fusion and ER cross sections are compared with the experimental values of the capture and ER cross sections of the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ and ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reactions in Figs. 3.5 and 3.6, respectively. The partial fusion cross sections are used as the input data in calculations of the ER cross sections by the advanced statistical model [64; pp. 064607-9]. It is seen from these figures that the theoretical results for the $3 n$-evaporation channel are in good agreement with the experimental data while the theoretical curve obtained for the 2 n -evaporation channel is in good agreement with the data up to energies $E_{C N}^{*}=30$ MeV and 28 MeV for the the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ and ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reactions, respectively.

## § 3.6 Conclusion for Chapter III

In this chapter results of cross section, for each stages of reaction, were obtained and compared with the experimental data. Partial capture cross section was taken as sum of fusion, quasifission, and fast fission cross sections. The evaporation residue cross sections after emission 2 and 3 neutrons is calculated by the use of $\left\langle\sigma_{\text {fus }}^{(\ell)}\left(E_{c . m}\right)\right\rangle$, taking into account the dependence of the fission barrier on the angular momentum.

The difference between observed cross sections of the ER of the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ and ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reactions formed in the 2 n and 3 n channels has been explained by two reasons related to the entrance channel characteristics of these reactions. The first reason is the difference in the sizes and position of the potential wells of the nucleus-nucleus interaction calculated for these reactions. The presence of two extra neutrons in isotope ${ }^{36} \mathrm{~S}$ and projectile-like fragments makes the potential well deeper and lower for the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction. Therefore, the capture cross section for this reaction is larger than the one of the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction, i.e. the larger number of DNS being to be transformed into compound nucleus is formed in the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction. The second reason is the difference in the heights of the intrinsic fusion barrier $B_{f u s}^{*}$ appearing on the fusion trajectory by nucleon transfer between nuclei of the DNS formed after capture. The value of $B_{f u s}^{*}$ calculated for the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction is higher than the one obtained for the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction. This fact is caused by the difference between the $N / Z$-ratios in the light fragments of the DNS formed during the capture in these reactions.

Presented results have been published in the following journals:

- Nasirov A.K., Kayumov B.M., Mandaglio G., Giardina G., Kim K. and Kim Y. The effect of the neutron and proton numbers ratio in colliding nuclei on the formation of the evaporation residues in the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ and ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reactions // European Physical Journal A. - Springer-SIF (Germany), 2019. Vol. 55 - pp. 29.


## IV. Angular distribution of reaction products

## § 4.1. Introduction

Angular distribution of the reaction products is one of important characteristics allowing to make conclusions about reaction mechanism which is responsible for their production. The correlation between mass and angular distributions of the binary fragments depends drastically on the values of the total kinetic energy loss [67; pp. 109-121]. Main result of this paper is the change of the angular distribution from the "bell shape" with the maximum for the projectile-like products to the monotonic shape distributed involving large range of the charge distribution by the increase of the energy loss. The strength of dissipation is determined by the orbital angular momentum. The increase of rotational energy decreases the amount of the being dissipated kinetic energy. The full-momentumtransfer reaction does not take place at the deep inelastic heavy ion collisions. In this case the relative motion of the colliding nuclei is not completely damped and the projectile-like and target-like products go away. The lifetime of DNS formed in deep inelastic collisions would be shorter than those of capture cases. This phenomena were well described in the review by Schröder W. U. and Huizenga J. R. in Ref. [68; pp. 115-726]. The dynamics of heavy-ion collisions is studied experimentally for metastable composite systems formed in the reactions with the projectiles of ${ }^{238} \mathrm{U}$ ions from the UNILAC accelerator of GSI (Darmstadt) on the ${ }^{48} \mathrm{Ca}$ and ${ }^{52} \mathrm{Ti}$ targets [69; pp. 157-183]. The experiments in the Australian National University's Heavy Ion Accelerator Facility and CUBE spectrometer extensive mass-angle distributions of the binary reaction products have been measured [70; pp. 054618-22]. The different mappings of mass-angle distribution characteristics (including timescales) show a systematic dependence on entrance channel and compound nucleus facilities. The authors of Ref. [70; pp. 054618-22] had concluded that results provide an empirical baseline to assess effects of nuclear structure and
entrance channel at lower beam energies. The analysis of these results can be motivation of the validation of complete dynamical models of heavy element fusion through comparison of mass-angle distributions. The similar studies were carried out there later for the ${ }^{40} \mathrm{Ca}+{ }^{186} \mathrm{~W},{ }^{192}$ Os reactions [71; pp. 034608-13]. The authors have made conclusion that the presence of a weak mass-asymmetric quasifission component attributed to the higher angular momentum events can be reproduced with a shorter average sticking time but longer mass-equilibration time constant.

Recently, the analysis in Refs. [72; pp. 5. 73; pp. 227-238] on the inversekinematics ISODEC experiment led the authors to a claim of the observation of a new reaction mechanism in the reaction of ${ }^{78} \mathrm{Kr}+{ }^{40} \mathrm{Ca}$ at $E / A=10 \mathrm{MeV}$. In this experiment, the energy and angular distributions of the binary reaction products of the collision have been measured and the velocity and mass distributions were reconstructed. It is clear that the observed yields of the binary products are related with the deep inelastic collisions, quasifission and fusion-fission processes. In this work, the last two processes, where full momentum transfer takes place, have been analyzed, and the events with the component of the velocity distribution in the range of $1.5 \mathrm{~cm} / \mathrm{ns}<\mathrm{v}_{\mathrm{rel}}<3.5 \mathrm{~cm} / \mathrm{ns}$ peaking at $\mathrm{v}_{\mathrm{rel}}=2.4 \mathrm{~cm} / \mathrm{ns}$ were explored. The relative velocity is the difference between velocities of the observed fragments, $\mathrm{v}_{\mathrm{rel}}=\left|\overleftarrow{\mathrm{v}}_{1}-\overleftarrow{\mathrm{v}}_{2}\right|$. The individual fragment velocity vectors $\left(\overleftarrow{\mathrm{v}}_{1}, \overleftarrow{\mathrm{v}}_{2}\right)$ and the corresponding momenta are used to determine the corresponding velocity components, parallel and perpendicular to the beam, in the rest frame of the emitter, i.e., the fission source.

Three phenomena observed in the analysis of the reaction products inspired the authors of Refs. [72; pp. 5. 73; pp. 227-238] to suggest a new interesting reaction mechanism called "shock-induced fission following fusion" in central collisions. These three phenomena are summarised as follows.

The first observation is that the velocity distribution in the beam direction $\left(\mathrm{V}_{\|}\right)$ is slightly deformed whereas the spectrum of transversal relative velocity $\left(\mathrm{v}_{\perp}\right)$ is
isotropic. This can be seen in Fig. 4.1 where the correlated fragment velocities parallel and perpendicular to the beam direction in the emitter's rest frame are shown. The presented results are obtained for the mass-symmetric fission fragments with fission-like relative velocities and are drawn based on Fig. 6 of Ref. [73; pp. 227-238] to demonstrate one of the main arguments of the authors of Refs. [72; pp. 5. 73; pp. 227-238] to state the observation of the so-called "shock-induced fission following fusion".


Fig. 4.1. Correlated fragment velocities parallel $\left(\mathrm{v}_{\|}\right)$and perpendicular $\left(\mathrm{v}_{\perp}\right)$ to the beam in the rest frame of the emitter. The velocity of the centre-of-mass system in the laboratory frame is subtracted and the bands are built based on Fig. 6 of Ref. [73; pp. 227-238].

The second observation is the unusual properties of the fragment angular distribution $d \sigma / d \Theta_{\mathrm{HFr}}$, which is strongly anisotropic, except for mass-symmetric events, and not symmetric at $90^{\circ}$ as shown in Fig. 8 of Ref. [73; pp. 227-238]. For asymmetric fission events, the heavier fragment is preferentially emitted in the forward direction in the centre-of-mass system. For symmetric events, where |( $A_{1}-$ $\left.A_{2}\right) /\left(A_{1}+A_{2}\right) \mid<0.1$ with $A_{1}$ and $A_{2}$ being the mass numbers of the fragments, the
distribution is not isotropic and has maxima both at forward and backward angles. This behaviour indicates a rather strong alignment of the fission axis in the beam direction and demonstrates the dominant dynamical character of the process. Clear asymmetric fission events were found to have a tendency that more massive projectile-like fragments proceed along the beam direction, which seems to be the memory of the initial mass and velocity distributions. The observation of the heavy projectile-like fragments beyond the light target-like ones in the reaction of ${ }^{78} \mathrm{Kr}$ $+{ }^{40} \mathrm{Ca}$ seems to be unusual and, in Refs. [72; pp. 5. 73; pp. 227-238], the idea of "shock-induced fission following fusion" was suggested, which was claimed to occur in central collisions.

As shown in Fig. 9 of Ref. [73; pp. 227-238], the Galilean-invariant velocity distributions of $\alpha$-particles emitted from the forward-moving (mass-symmetric) fission fragments were observed to be isotropic. This means that the spin angular momenta of the emitting fragments are negligibly small, which supports the conclusion that the "shock-induced fission" occurs at small initial angular momentum ( $\ell \approx 0-40 \hbar$ ). The possibility of the transparency of the light target ${ }^{40} \mathrm{Ca}$ through heavy projectile ${ }^{78} \mathrm{~K}$ in central fusion-type heavy-ion collisions has been demonstrated by the presentation of plots of the density contours of projectileand target-like fragments in central ${ }^{78} \mathrm{Kr}+{ }^{40} \mathrm{Ca}$ collisions as a function of time in Fig. 3 in Ref. [73; pp. 227-238]. The small values of the spin of the binary emitting fragments are populated at the large orbital angular momenta $L$ since its dissipation leading to the increase of the fragments spin is weak due to smallness of the nuclear densities at large impact parameters of the entrance channel. This point is not in contradictions with the results of our work [74; pp. 89-103].

## §4.2 Angular distribution of DIC and quasifission products

From theoretical point of view, connection between expressions for angular distribution in the laboratory and center-of-mass systems can established easily:

$$
\begin{equation*}
\tan \theta_{1}=\frac{\sin \theta}{\frac{M_{1}}{M_{2}}+\cos \theta} \tag{4.1}
\end{equation*}
$$

or

$$
\begin{equation*}
\cos \theta_{1}=\frac{\frac{M_{1}}{M_{2}}+\cos \theta}{\left[1+2 \frac{M_{1}}{M_{2}} \cos \theta+\left(\frac{M_{1}}{M_{2}}\right)^{2}\right]^{1 / 2}} \tag{4.2}
\end{equation*}
$$

where $\theta_{1}$ and $\theta$ is the deviation angle of the projectile-like nuclei from the beam direction in the the laboratory and center-of-mass systems, respectively [75; pp. 501].

Due to attractive forces at small distances between nuclei, we can observe deviation to the negative angles of the reaction products relative to the beam direction as shown in Fig. 4.2. As a result, the products which are registered by the detector at the definite angle, can have different total kinetic energy distributions with the two maxima values corresponding to the closer and roundabout trajectories.


Fig. 4.2. Explanation of the appearance of two bumps $E_{1}$ and $E_{2}$ in the total kinetic energy distribution of the binary products observed at the laboratory system angle $\theta$ in heavy ion collisions.

The angular distribution of the projectile-like nuclei in the center-of-mass systems can be find from the one in the laboratory system coordinates by expression:

$$
\begin{equation*}
\cos \theta=M_{1} / M_{2} \sin ^{2} \theta_{1}+\left[\cos ^{2} \theta_{1}-\left(M_{1} / M_{2}\right)^{2} \sin ^{2} \theta_{1}\right]^{1 / 2} \tag{4.3}
\end{equation*}
$$

The mass and charge distributions of the DNS fragments depend on the nuclear structure and collision dynamics. The decay probability of system depends on its pre-scission barrier $B_{q f}$, which is determined by the depth of the potential well (see Fig. 2.3). The size of the potential well depends on the mass and charge asymmetry of DNS. The knowledge of lifetime of system $\tau_{\text {DNS }}$ and initial value of the orbital angular momentum $L$ is required to estimate the angular distribution of the reaction products. At the same time, $\tau_{\text {DNS }}$ depends on the maximum value of the charge and mass distribution, which is a function on time and peculiarities of the nuclear shell structure.

The rotational angle of DNS during capture for the given initial values of beam energy and orbital angular momentum $L_{0}$ is found by solving the equation of motions (1.7-1.11) for capture. If we neglect the decrease of the angular momentum of the dinuclear system by emission of light particles (gamma quanta, neutrons, etc.) during its evolution to quasifission, its angular momentum $L_{Z}$ can be considered as a constant value. We should note that $L_{Z}$ is less than the initial orbital angular momentum $L_{0}$ due to dissipation during the capture. Knowing values of angular momentum $L_{Z}$ and moment of inertia $J_{Z}$ for the DNS allows us to find its angular velocity $\Omega_{\text {DNS }}$. At the considered beam energies, the dinuclear system is formed when the interacting nuclei are trapped into potential well because the relative kinetic energy decreases due to the dissipation and it becomes not enough to overcome the quasifission barrier by the classical dynamical way (see Chapter I). The characteristic lifetime of DNS at quasifission is about or more than $5 \cdot 10^{-22} \mathrm{~s}$.

Once the angular momentum $L_{\text {DNS }}$ and moment of inertia $J_{\text {DNS }}$ of the dinuclear system are known, its angular velocity is obtained as $\Omega_{\mathrm{DNS}}=L_{\mathrm{DNS}} / J_{\mathrm{DNS}}$.

To find the angular distribution of the quasifission fragments, we estimate the rotational angle $\theta_{\text {DNS }}$ at the break-up of the system as

$$
\begin{equation*}
\theta_{\mathrm{DNS}}=\theta_{\text {in }}+\Omega_{\mathrm{DNS}} \cdot \tau_{\mathrm{DNS}}, \tag{4.4}
\end{equation*}
$$

where $\theta_{\text {in }}$ is determined by the dynamical calculations of Eqs. (1.9) and (1.10) for the entrance channel of the reaction, i.e., at the capture stage. The value of $\theta_{\text {in }}$ depends on the angular momentum and orientation angles $\alpha_{1}$ and $\alpha_{2}$ of the symmetric axis of the colliding nuclei at a given $E_{c . m}$. The lifetime of the DNS configuration $\tau_{\text {DNS }}$ with $Z=Z_{1}$ and $Z_{2}=Z_{C N}-Z$, where $Z_{C N}$ is the charge number of the compound nucleus, is determined by the quasifission barrier $B_{q f}$ and the excitation energy $E_{Z}^{*}$ for given values of beam energy and angular momentum $\ell$ through

$$
\begin{equation*}
\tau_{D N S}=\frac{\hbar}{\Lambda_{Z}^{q f}}, \tag{4.5}
\end{equation*}
$$

where the decay width of the DNS is given by equation (2.8).

## § 4.3 Angular distribution of deep inelastic collision and quasifission products

The angular distribution of the reaction products is obtained by calculating the rotational angle with the lifetime and angular velocity of the DNS determined by Eq. (4.4). In Fig. 4.3 we present the results for the rotational angle of the DNS formed in the reaction of ${ }^{78} \mathrm{Kr}+{ }^{40} \mathrm{Ca}$ as a function of orbital angular momentum for several values of the initial energy. It can be seen that, in the middle values $(40 \hbar-70 \hbar)$ of orbital angular momentum, the rotational angle of the DNS is larger and the maximum value of the rotational angle is close to $180^{\circ}$. This means that the lifetime and rotational velocity of the DNS allow the projectile-like fragment to go beyond
the target fragment. The smallness of the probability of the DNS decay in the perpendicular direction with respect to the beam direction with $L=40 \hbar-70 \hbar$ may be understood from Fig. 4.4. Certainly the energy accumulated in the rotation of the DNS will increase the relative velocity of the decay products in the forward and backward directions to the beam. But the rotational energy contributing to the increase of the relative velocity of the decay products in the perpendicular direction would be small since the corresponding values of the angular momentum are small such as $L=10-30 \hbar$ (see Fig. 4.4).


Fig. 4.3. The rotational angle of the DNS formed in the ${ }^{78} \mathrm{Kr}+{ }^{40} \mathrm{Ca}$ reaction as a function of orbital angular momentum $\ell$ for a given $E_{l a b}$.

This phenomenon is observed in heavy ion collisions even with massive nuclei. For example, in Ref. [76; pp. 227-232], the authors discussed the emission of the target-like nucleus in the beam direction of the laboratory system but with a velocity smaller than that of the compound nucleus. The intensity of the low-velocity peak was found to be much lower than that of the high-velocity peak. The two-peak structure was observed for all Rn, Fr and Ra isotopes, while it was found to fade for Po and At [76; pp. 227-232].

Combined with Fig. 4.3, Fig. 4.4 illustrates the dependence of the rotational
angle of the DNS formed in the ${ }^{78} \mathrm{Kr}+{ }^{40} \mathrm{Ca}$ reaction with angular momentum $\ell$ for the initial energy from 760 MeV to 800 MeV . It is found that, in the collision with the initial values of $L=50 \hbar$ and $E_{l a b}=770 \mathrm{MeV}$, the rotational angle of the DNS has the maximum value that corresponds to the situation when the projectile and target nuclei exchange their positions relative to the beam direction. Then, after the decay of the DNS, the projectile-like product can be observed in the forward hemisphere with a speed larger than that of the compound nucleus due to the repulsion by the Coulomb force of the target-like products. This phenomenon is


Fig. 4.4. Contour diagram for the rotational angle of the dinuclear system formed in the ${ }^{78} \mathrm{Kr}+{ }^{40} \mathrm{Ca}$ reaction as a function of the angular momentum $\ell$ and energy

$$
E_{l a b} .
$$

consistent with the observation discussed in Ref. [72; pp. 5]. The relative velocity of these fragments is in the range of $2.4-2.7 \mathrm{~cm} / \mathrm{ns}$, which overlaps with the experimental data presented in Fig. 2 of Ref. [72; pp. 5], where the yield of binary fragments flying in the opposite direction, i.e., $-1.0<\cos (\alpha)<-0.7$ with $\alpha$ being the folding angle between the centre-of-mass velocities of the two fragments was discussed. This observation was interpreted in Ref. [72; pp. 5] as a new reaction
mechanism of a prompt shock-induced fission following the fusion of ${ }^{78} \mathrm{Kr}+{ }^{40} \mathrm{Ca}$ nuclei.

Therefore, through the presented results, we suggest another mechanism of quasifission producing massive products in the forward hemisphere in capture reactions. The products formed through this mechanism can contribute to the yield of the fragments observed in Ref. [72; pp. 5]. We also find that the rotational velocities of the reaction products around their own axes are very small, and certainly the alpha particles emitted from these products after quasifission are expected to be distributed isotropically if the intrinsic spin of the product which emits $\alpha$ particles is small.

## § 4.4 Conclusion for Chapter IV

In summary, performed theoretical study on the angular and mass distributions of quasifission fragments in the reaction of ${ }^{78} \mathrm{Kr}+{ }^{40} \mathrm{Ca}$ indicates that the rotational angle of the dinuclear system can reach $180^{\circ}$ at collisions with relative angular momentum of $L=(40-60) \hbar$ (can be seen in fig. 4.3 and 4.4). As a result, the projectile-like products can be observed in the forward hemisphere with a velocity in the range of $2.4-2.7 \mathrm{~cm} / \mathrm{ns}$, which is consistent with the experimental observations reported in Refs. [72; pp. 5. 73; pp. 227-238].

In the experiment of Ref.[72; pp. 5], the emission of alpha-particles was also found to be nearly isotropic being emitted from the projectile-like products in the forward hemisphere. In the present work, the quasifission mechanism can reproduce the observed angular and mass distributions of these projectile-like products. The energy accumulated due to the rotation of the DNS increases the relative velocity of the decay products in the forward and backward directions since a relatively large value of angular momentum, namely, $L=(40-60) \hbar$, allows DNS decays in these directions. The rotational energy contributing to the increase of the relative velocity of the decay products in the perpendicular direction is, however, small due to the small value of the corresponding angular momentum, $L=(10-30) \hbar$. As a result, the
velocity distribution of the fission-like products observed in the experiment of Refs. [72; pp. 5. 73; pp. 227-238] can have a slightly elongated shape along the beam direction.

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- Nasirov A.K., Kayumov B.M. and Yongseok Oh Peculiarities of quasifission reactions in heavy ion collisions // Nuclear Physics A. - Elsevier (Netherland), 2016. - Vol. 946 - pp. 89 -103.


## Main results and Conclusion

The main results of the theoretical research, which was carried out on the theme of the PhD dissertation "Dynamics of interaction in heavy ions collisions at the energy near to coulomb barrier" has leaded to the following conclusions:

1. It was shown that, the capture of incoming projectile-nucleus by targetnucleus is necessary condition for their complete fusion. The probability of capture depends on the size of the potential well formed by the sum of the Coulomb potential and nuclear attraction. It is clearly seen that the size of the potential well decreases with increasing values of the orbital angular momentum of a collision. The presence of two extra neutrons in isotope ${ }^{36} \mathrm{~S}$ and projectile-like fragments makes the potential well deeper and lower for the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction. Therefore, the capture cross section for this reaction is larger than the one of the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction, i.e. the larger number of DNS being to be transformed into compound nucleus is formed in the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction. This increases the probability of complete fusion.
2. By the evolution of the DNS, the probability of the complete fusion depends on the internal fusion barrier $B_{f u s}^{*}$, which is determined by the structure of the driving potential. For heavy systems appears high fusion barrier, which reduces the probability of fusion. The difference in the heights of the intrinsic fusion barrier $B_{f u s}^{*}$ appearing on the fusion trajectory by nucleon transfer between nuclei of the DNS formed after capture. The value of $B_{f u s}^{*}$ calculated for the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction is higher than the one obtained for the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction. This is seen from the comparison of the shape of the driving potentials and landscape of PES, which are calculated for these two reactions. This fact is caused by the difference between the N/Z-ratios in the light fragments of the DNS formed during the capture in these reactions.
3. It was reached that, the difference between observed cross sections of the evaporation residue of the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ and ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reactions formed in the

2 n and 3 n channels has been explained by two reasons related to the entrance channel characteristics of these reactions. The first reason is the difference in the sizes and position of the potential wells of the nucleus-nucleus interaction calculated for these reactions. The second reason is the difference in the heights of the intrinsic fusion barrier $B_{\text {fus }}^{*}$ appearing on the fusion trajectory by nucleon transfer between nuclei of the DNS formed after capture. Also, a larger hindrance to complete fusion in the reaction with ${ }^{34} \mathrm{~S}$ can be observed from the analysis of the yield of the projectile-like capture products. The intense yield of the projectile-like capture products decreases the number of events going to complete fusion which produces fusion-fission products and evaporation residues after emission neutrons and light charged particles. Due to these consequences, the use of the neutron rich isotope ${ }^{36}$ S makes the ER cross section larger in the ${ }^{36} \mathrm{~S}+{ }^{206} \mathrm{~Pb}$ reaction at the de-excitation of compound nucleus in comparison with the ones of the ${ }^{34} \mathrm{~S}+{ }^{208} \mathrm{~Pb}$ reaction.
4. It was obtained that, the observed yields of the binary products in ${ }^{78} \mathrm{Kr}(10$ $\mathrm{A} / \mathrm{MeV})+{ }^{40} \mathrm{Ca}$ reaction are related quasifission and fusion-fission processes. It can be seen from the results, that the shape of the charge and mass distributions of the quasifission process depends on the orbital angular momentum. In collisions with $L<60 \hbar$ the average values of the charge and mass distributions are rather concentrated near the projectile/target masses and charges at around $\left(Z_{L}=18, A_{L}=38\right)$ for the lighter product and at around $\left(Z_{H}=38, A_{H}=78\right)$ for the heavier product, which has a good agreement with the experimental results. In collisions with $60 \hbar<L<80 \hbar$ the charge and mass distributions extend up to the mass symmetric region overlapping with those of the fusion-fission products.
5. Moreover, measuring the rotational angle of the dinuclear system in the experiment allow us to establish a life time of DNS. From theoretical study on the angular distributions of quasifission fragments in the reaction of ${ }^{78} \mathrm{Kr}$ $(10 \mathrm{~A} / \mathrm{MeV})+{ }^{40} \mathrm{Ca}$, which indicates that the rotational angle of the dinuclear system can reach $180^{\circ}$ at collisions with relative angular momentum of $L$
$=(40-60) \hbar$. As a result, the projectile-like products can be observed in the forward hemisphere with a velocity in the range of $2.4-2.7 \mathrm{~cm} / \mathrm{ns}$, which is consistent with the experimental observations.

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